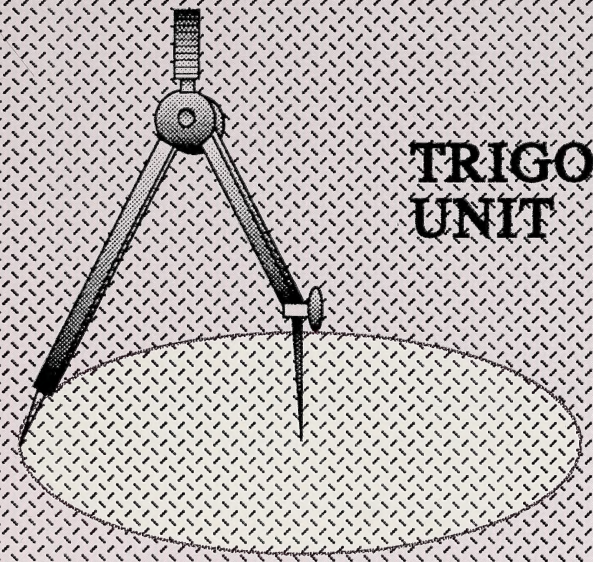
MATHEMATICS 23



TRIGONOMETRY
UNIT 7

Distance Learning

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Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

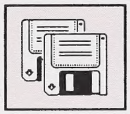
An explanation of what they mean is written beside each visual cue.

Key Idea
- flagging important ideas

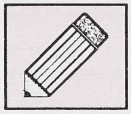
Audiotape
- learning by listening to an audiotape

What You Already Know
- reviewing what you already know

Another View
- exploring different perspectives



Computer Software
- learning by using computer software



Review
- studying previous concepts

Solutions
- correcting the activities

Videotape
- learning by viewing a videotape

Introduction
- introducing the unit

Extra Help
- providing additional study

Print Pathway
- choosing a print alternative

What Lies Ahead
- previewing the unit

Extensions
- going on with the topic

Calculator
- using your calculator

Exploring the Topic
- actively learning new concepts

What You Have Learned
- summarizing what you have learned

Mathematics 23

Course Overview

Mathematics 23 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Powers and Radicals	10%
Unit 2 Algebra	12%
Unit 3 Mathematics of Finance	4%
Unit 4 Linear Relations	12%
Unit 5 Systems of Equations	16%
Unit 6 Geometry	16%
Unit 7 **Trigonometry**	**16%**
Unit 8 Statistics	14%
	100%

Unit Assessment

After completing the unit you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal your teacher will determine what this assessment will be. It may be

Unit assignment - 50%
Supervised unit test - 50%

Introduction to Trigonometry

This unit covers topics dealing with trigonometry. Each topic contains explanations, examples, and activities to assist you in understanding trigonometry. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the Solutions in **Appendix A**. In several cases there is more than one way to do the question.

Unit 7 Trigonometry

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	• What Lies Ahead	• Extensions	
	• Exploring Topic 1		
10%	**Topic 2:** Similar Triangles		32
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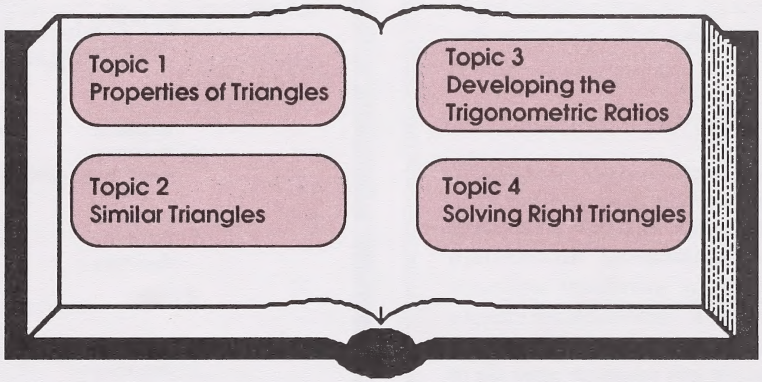
Trigonometry

Everyone, at one time or another, wishes that there were better ways to solve problems in everyday life. Mathematicians are not any different. They are always looking for easier or better ways to solve various situations. Trigonometry is a branch of mathematics which allows people, through the use of triangles, to measure distances that are difficult or even impossible to determine otherwise. Such measurement is often called indirect measurement. It would be reasonable to conclude that surveyors, navigators, and astronomers make extensive use of trigonometry. However, these are certainly not the only individuals who take advantage of this branch of mathematics.

Unit 7 Trigonometry

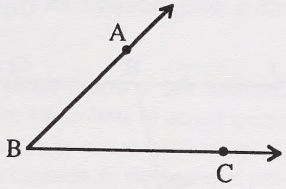


What You Already Know

Refresh your memory!

The following are the most important facts that must be kept in mind as you work through this unit.

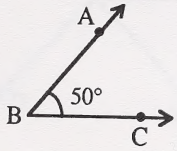
- An **angle** may be named using three letters or just the letter at its vertex.



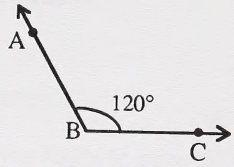
This angle is made up of two rays which are $\overrightarrow{BA}$ and $\overrightarrow{BC}$. B is the vertex of the angle.

This is $\angle ABC$ or $\angle B$. It can also be named as $\angle CBA$. The vertex is always placed between the other two letters.

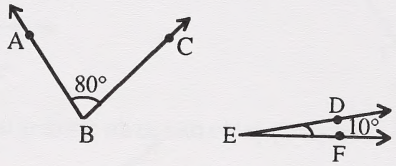
- An **acute angle** has a measure between 0° and 90°.



- An **obtuse angle** has a measure between 90° and 180°.

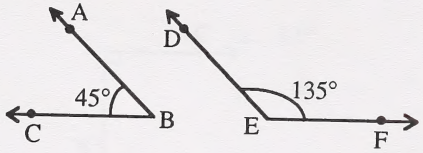


- **Complementary angles** are two angles whose total measure is 90°.



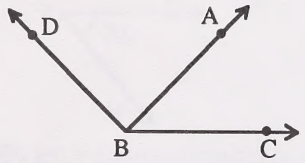
$\angle B + \angle E = 80° + 10°$
$= 90°$

- **Supplementary angles** are two angles whose total measure is 180°.



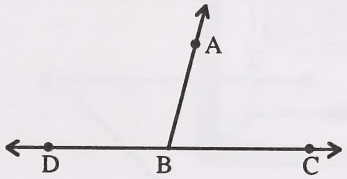
$\angle B + \angle E = 45° + 135°$
$= 180°$

- **Adjacent angles** are two angles which have a common side or ray and a common vertex.

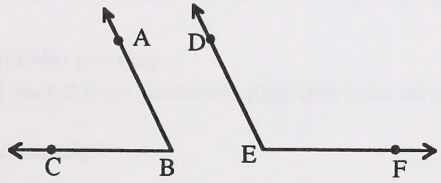


∠ABC and ∠ABD are adjacent angles, since they have $\overrightarrow{BA}$ in common and vertex B in common.

- **Linear angles** are two angles which add up to 180° or which form a straight line. Linear angles may be adjacent or separate.

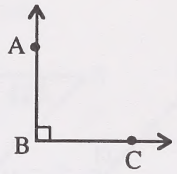


∠ABC and ∠ABD are adjacent linear angles.



∠ABC and ∠DEF are nonadjacent linear angles.

- The sum of the three angles of any triangle is 180°.
- A **right angle** is an angle which measures 90°.



∠ABC is a right angle. Instead of using right angle ABC, use right ∠ABC.

A square box symbol is used to show which angle is a right ∠ or where the right ∠ is.

- A **proportion** is made up of two equal fractions or ratios.

For example,

$$\frac{4}{7} = \frac{8}{14}$$

- A proportion which has a variable can be solved by cross multiplication. The cross multiplication is often referred to as finding the cross products.

For example,

$$\frac{5}{n}=\frac{30}{36}$$
$$30\times n=36\times 5$$
$$30n=180$$
$$\frac{30n}{30}=\frac{180}{30}$$
$$n=6$$

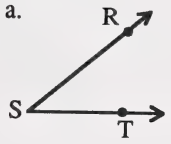
Do the exercises which follow to see how well you can apply your knowledge of these basic facts to actual questions. It is a good idea to show all your work as you work through the questions so you can compare your answers to the answers provided.

Review

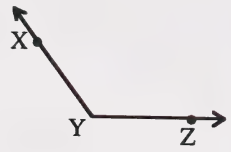
Try the following questions.

1. Name the following angles in three different ways.

a.



b.



c.

2. Classify each of these angles as acute, obtuse, or right.

a. 5°

b. 179°

c. 90°

d. 83°

e. 98°

f. 14°

g. 125°

3. Give the complementary angle for each of the following.

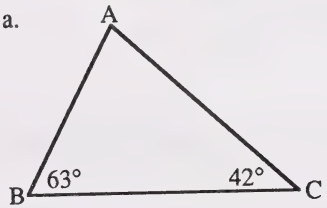
   a. 63° b. 37° c. 71°

4. Give the supplementary angle for each of the following.

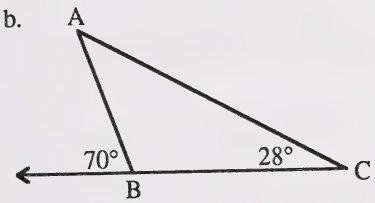
   a. 112° b. 153° c. 96°

5. Find the measure of the missing angles in each sketch provided.

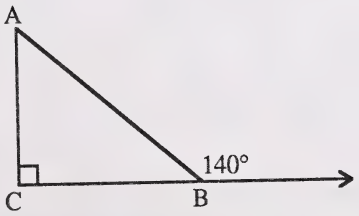
   a.



   b.



   c.



6. Solve for the unknown in each proportion given. Check your answers. Show all your work.

   a. $\frac{2}{3}=\frac{n}{15}$ b. $\frac{n}{12}=\frac{3}{9}$

   c. $\frac{12}{13}=\frac{36}{n}$ d. $\frac{5}{n}=\frac{15}{18}$

   e. $\frac{n}{6}=\frac{5}{25}$ f. $\frac{3\frac{1}{2}}{n}=\frac{6\frac{1}{4}}{10\frac{1}{2}}$

   g. $\frac{10.2}{12.6}=\frac{n}{4}$ h. $\frac{5.25}{3\frac{1}{2}}=\frac{4.75}{n}$

   i. $\frac{n}{6}=\frac{9\frac{1}{3}}{14.5}$

Now go to the **Review** solutions in **Appendix A**.

If you had difficulty doing these problems, particularly the ones pertaining to proportions, you may wish to study ratios and proportions in more detail in Practical Mathematics, Module 4.

Topic 1 Properties of Triangles

Introduction

Did you ever consider what life would be like if you did not have the skills to build and use words? How would you communicate if you were not able to build and use sentences? All that you wished to pass on to a friend would have to be done by sign language. Using words and sentences, ideas can be put into spoken or written form. To be well versed in trigonometry, you need to know the terms related to this branch of mathematics. In this section and the ones to follow, you will learn how to apply many new terms related to triangles and to trigonometry.

What Lies Ahead

Throughout the topic you will learn to

1. label the vertices and sides of a triangle
2. find the measure of the third angle of a triangle
3. label the sides of a right triangle as opposite, adjacent, and hypotenuse with respect to a given or chosen angle
4. use the Theorem of Pythagoras

Now that you know what to expect, turn the page to begin your study of properties of triangles.

Exploring Topic 1

Activity 1

Label the vertices and sides of a triangle.

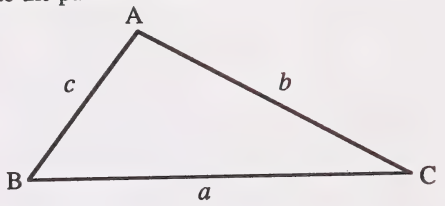
A triangle is a polygon which has three sides and three angles. You will find that when studying triangles and when working with triangles, direct reference must be made to a particular triangle. This means that triangles have to be named.

Any triangle, just like any other geometric figure, is named using upper case or capital letters. The points at which the sides of a triangle meet are called vertices. The sides of a triangle are line segments and are named using lower case or small letters corresponding to the vertex which is opposite that particular side. $\angle A$ is opposite side a. $\angle B$ is opposite side b. $\angle C$ is opposite side c. Sides can also be named using two letters and the symbol for a line segment. For example, in ΔABC the sides are $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$.

Study the example which follows.

Example 1

Name the parts of ΔABC.



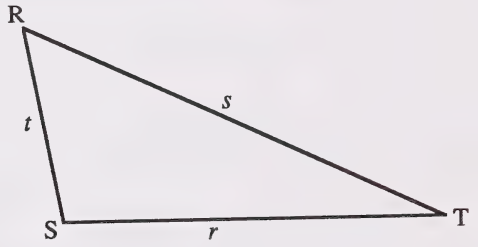
Solution:

The three angles are $\angle ABC$ or $\angle B$; $\angle ACB$ or $\angle C$; and $\angle BAC$ or $\angle A$;

The three sides are $\overline{AB}$ or c; $\overline{AC}$ or b; and $\overline{BC}$ or a.

Example 2

Name the parts of ΔRST.



Solution:

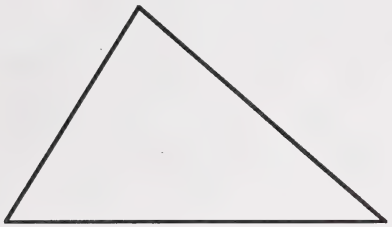
The three angles are $\angle RST$ or $\angle TSR$ or $\angle S$; $\angle RTS$ or $\angle STR$ or $\angle T$; and $\angle SRT$ or $\angle TRS$ or $\angle R$.

The three sides are $\overline{RS}$ or $\overline{SR}$ or t; $\overline{ST}$ or $\overline{TS}$ or r; $\overline{RT}$ or $\overline{TR}$ or s.

Do any two of the questions which follow.

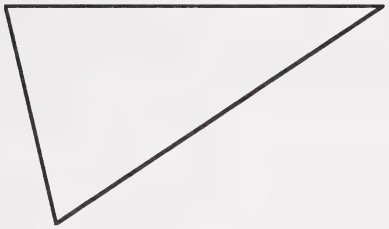
If you need additional practice, do all of the questions.

1.



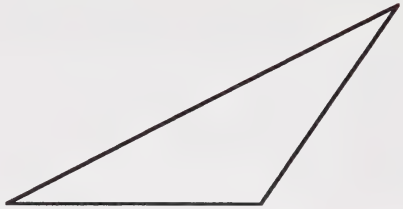
a. Use D, E, and F to name the triangle.
b. Name the angles of the triangle in three different ways.
c. Name the sides of the triangle in three different ways.

2.



a. Use O, P, and Q to name the triangle.
b. Name the angles of the triangle in three different ways.
c. Name the sides of the triangle in three different ways.

3.



a. Use M, A, and N to name the triangle.
b. Name the angles of the triangle in three different ways.
c. Name the sides of the triangle in three different ways.

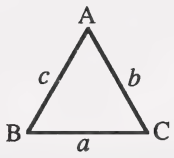
For solutions to **Activity 1**, turn to **Appendix A, Topic 1**.

Activity 2

Find the measure of the third angle of a triangle.

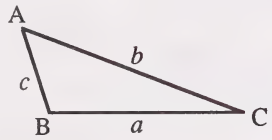
It is an established geometric fact that the sum of the three angles of any triangle is 180°. This situation is the same for any triangle of any size. This applies to the following triangles.

- an acute triangle



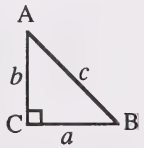
$\angle A + \angle B + \angle C = 180°$

- an obtuse triangle



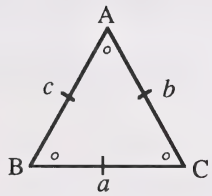
$\angle A + \angle B + \angle C = 180°$

- a right triangle



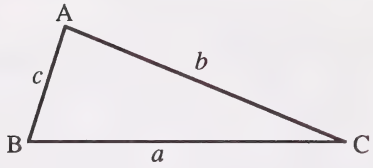
$\angle A + \angle B + \angle C = 180°$

- an equiangular or equilateral triangle



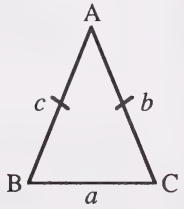
$\angle A + \angle B + \angle C = 180°$

- a scalene triangle



$\angle A + \angle B + \angle C = 180°$

- an isosceles triangle



$\angle A + \angle B + \angle C = 180°$

It is sometimes necessary to find the measure of a third angle when the measures of two angles are given. Study the following example.

Equiangular triangle: all angles are equal in measure.

Equilateral triangle: all sides are equal in measure.

Scalene triangle: all sides are different in measure.

Isosceles triangle: two sides are equal in measure.

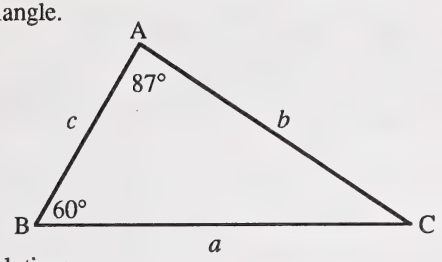
Acute triangle: all angles measure less than 90°.

Obtuse triangle: one angle measures more than 90°.

Right triangle: one angle measures 90°.

Example 3

Find the measure of the unknown angle in each triangle.

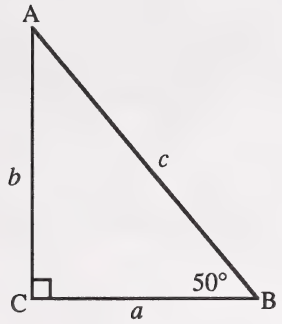


Solution:

$\angle C = 180° - (60° + 87°)$

$\angle C = 180° - 147°$

$\angle C = 33°$

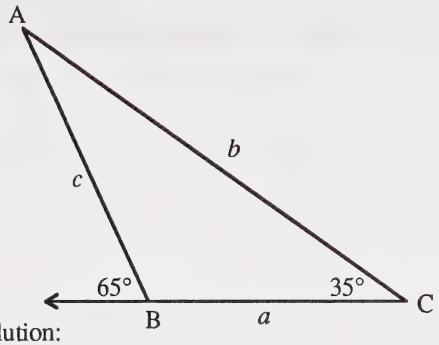


Solution:

$\angle A = 180° - (90° + 50°)$

$\angle A = 180° - 140°$

$\angle A = 40°$



Solution:

$\angle ABC = 180° - 65°$

$\angle ABC = 115°$

$\angle A = 180° - (115° + 35°)$

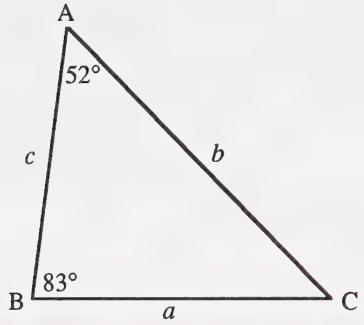
$\angle A = 180° - 150°$

$\angle A = 30°$

Do any two of the following three questions. If you need additional practice, do the rest of the questions.

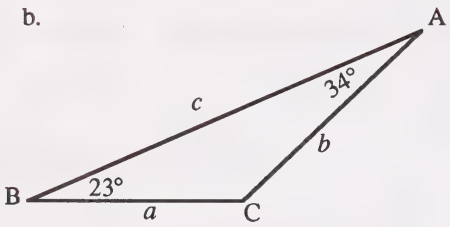
1. Find the measure of the unknown angle.

   a.



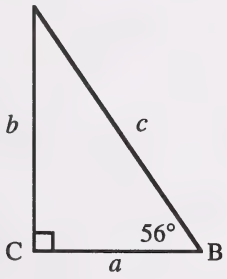
This symbol stands for a right angle which measures 90°.

b.

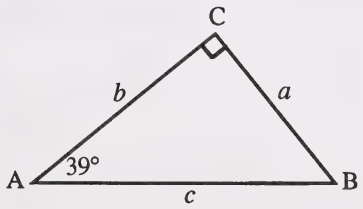


2. Find the measure of the unknown angle.

a.

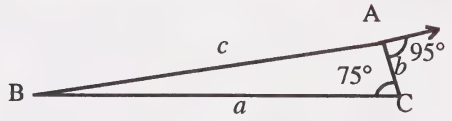


b.

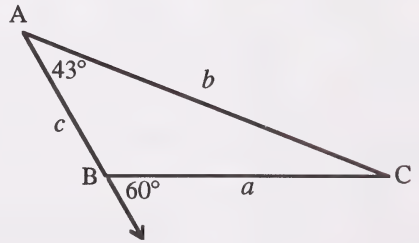


3. Find the measures of the unknown angles.

a.



b.



For solutions to **Activity 2**, turn to **Appendix A, Topic 1**.

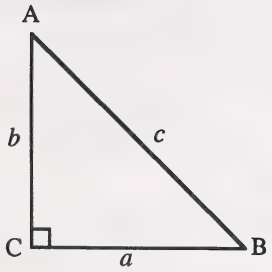
Activity 3

Label the sides of a right triangle as opposite, adjacent, and hypotenuse with respect to a given or chosen angle.

Besides being able to name the parts of a triangle properly, you must also be able to identify the side opposite a chosen angle and the side adjacent to a particular angle. It is equally important to select the hypotenuse of a right triangle. Study the example which follows.

Example 4

Use A, B, and C to label the triangle. Name the sides opposite the two acute angles. Also, tell which side is the hypotenuse.



Solution:

The two acute angles are $\angle A$ and $\angle B$.

The side opposite $\angle A$ is $\overline{BC}$ or a.

The side adjacent to $\angle A$ is $\overline{AC}$ or b.

The side opposite $\angle B$ is $\overline{AC}$ or b.

The side adjacent to $\angle B$ is $\overline{BC}$ or a.

The hypotenuse is the longest side of any right triangle, or it is the side opposite the right angle.

The hypotenuse in ΔABC is $\overline{AB}$ or c.

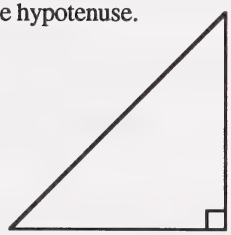
Try the following exercises.

Do either question 1 or 2.

1. a. Use C, D, and E to label the given triangle.

   b. For each of the two acute angles, name the side opposite each and the side adjacent to each.

   c. Identify the hypotenuse.

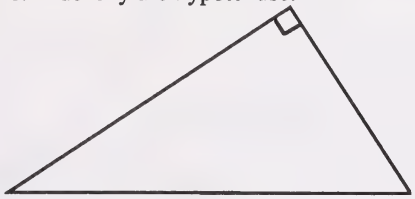


An acute angle measures less than 90°.

2. a. Use X, Y, and Z to label the given triangle.

   b. Choose the two acute angles, and name the side opposite each and the side adjacent to each.

   c. Identify the hypotenuse.



For solutions to **Activity 3**, turn to **Appendix A, Topic 1**.

Activity 4

Use the Theorem of Pythagoras.

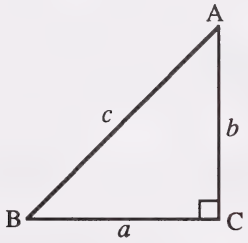
From your previous study of mathematics you probably remember that there exists a special relationship between the measures of the sides of a right triangle. This special relationship is called the Theorem of Pythagoras. This special situation was discovered by Pythagoras who was a famous Greek mathematician.

He found that if the measure of the hypotenuse is squared, the resulting value is equal to the sum of the squares of the measures of the other two sides.

This rule can be stated as follows.

For right triangle ABC, where $\angle C = 90°$,

$c^2 = a^2 + b^2$ or $a^2 + b^2 = c^2$.

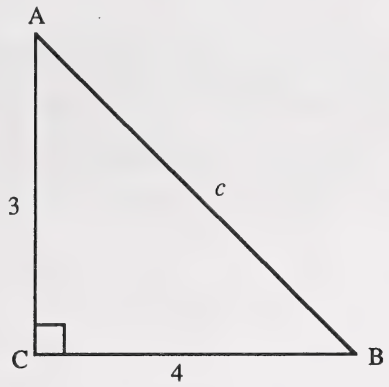


Using this rule, you can find the missing values for a right triangle as shown in the example that follows.

Adaptations of the Theorem of Pythagoras, $c^2 = a^2 + b^2$, can be written as $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.

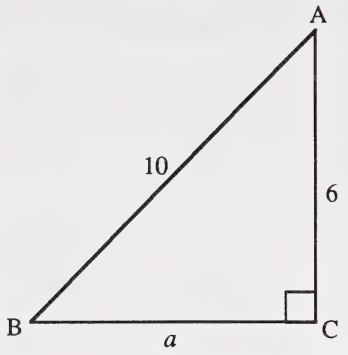
Example 5

Find the measure of the unknown side in each of the following right triangles.



Solution:

$$c^2 = a^2 + b^2$$
$$c^2 = 4^2 + 3^2$$
$$c^2 = 16 + 9$$
$$c^2 = 25$$
$$\sqrt{c^2} = \sqrt{25}$$
$$c = 5$$

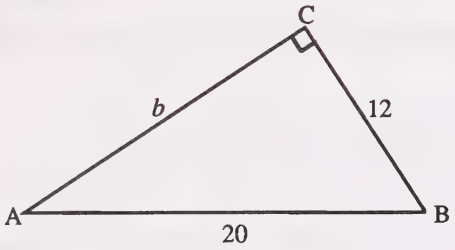


Solution:

$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$
$$a^2 = 10^2 - 6^2$$
$$a^2 = 100 - 36$$
$$a^2 = 64$$
$$\sqrt{a^2} = \sqrt{64}$$
$$a = 8$$

$\sqrt{c^2}$ means the square root of c^2 which is c, and $\sqrt{25}$ means the square root of 25 which is 5.

$\sqrt{a^2}$ means the square root of a^2 which is a, and $\sqrt{64}$ means the square root of 64 which is 8.



Solution:

$$c^2 = a^2 + b^2$$
$$b^2 = c^2 - a^2$$
$$b^2 = 20^2 - 12^2$$
$$b^2 = 400 - 144$$
$$b^2 = 256$$
$$\sqrt{b^2} = \sqrt{256}$$
$$b = 16$$

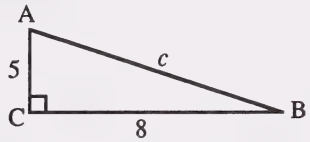
In the previous examples, all the values worked out evenly to whole numbers. This does not have to be the case in all situations. When the numbers are not perfect squares, rounding off the answer is required. Notice what happens in the following example.

Example 6

Find the measure for the missing values in each triangle.

Round the answers to

a. the nearest whole number
b. the nearest tenth
c. the nearest hundredth
d. the nearest thousandth



Solution:

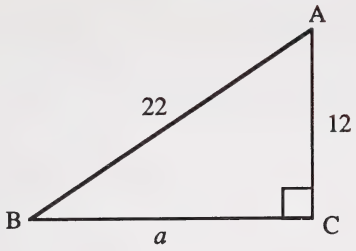
$$c^2 = a^2 + b^2$$
$$c^2 = 8^2 + 5^2$$
$$c^2 = 64 + 25$$
$$c^2 = 89$$
$$\sqrt{c^2} = \sqrt{89}$$
$$c \doteq 9.43398$$

a. 9
b. 9.4
c. 9.43
d. 9.434

$\sqrt{b^2}$ means the square root of b^2 which is b, and $\sqrt{256}$ means the square root of 256 which is 16.

Numbers such as 25, 64, and 400 are called perfect squares. The following are other perfect square values.
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, . . .

14.6375 rounded to the nearest whole number is 15; to the nearest tenth or one decimal place is 14.6; to the nearest hundredth or two decimal places is 14.64; and to the nearest thousandth or three decimal places is 14.638.

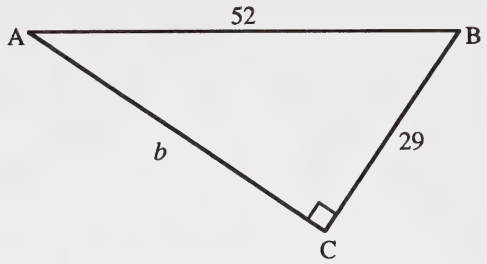


Solution:

$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$
$$a^2 = 22^2 - 12^2$$
$$a^2 = 484 - 144$$
$$a^2 = 340$$
$$\sqrt{a^2} = \sqrt{340}$$
$$a \doteq 18.439\,09$$

a. 18 b. 18.4

c. 18.44 d. 18.439



Solution:

$$c^2 = a^2 + b^2$$
$$b^2 = c^2 - a^2$$
$$b^2 = 52^2 - 29^2$$
$$b^2 = 2704 - 841$$
$$b^2 = 1863$$
$$\sqrt{b^2} = \sqrt{1863}$$
$$b \doteq 43.162\,48$$

a. 43 b. 43.2

c. 43.16 d. 43.162

The Theorem of Pythagoras is often used to solve practical problems. Study the example provided.

Example 7

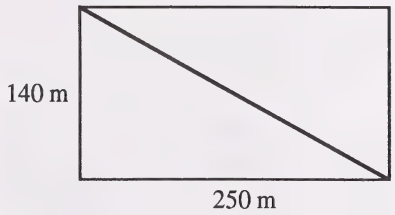
On the way to school, Sam chose to cross a rectangular field by going from corner to corner or diagonally. The field was 250 m long and 140 m wide. How much shorter was Sam's chosen path instead of going along the edges of the field? Round the answer to the nearest metre.

The distance across the field was 287 m. If he walked along the edges, he would go 390 m. By crossing diagonally, Sam would have covered 103 m less when going to school.

Try the questions that follow.

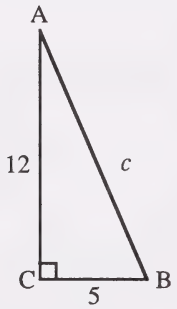
Do either the odd-numbered or the even-numbered questions of 1 through 6.

Solution:

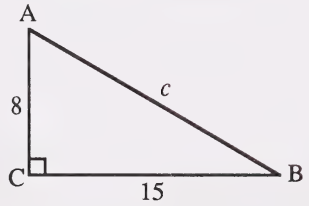


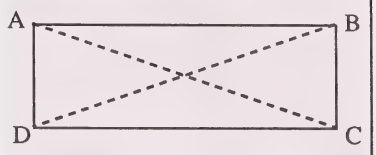
$$c^2 = a^2 + b^2$$
$$c^2 = (250)^2 + (140)^2$$
$$c^2 = 62\,500 + 19\,600$$
$$c^2 = 82\,100$$
$$\sqrt{c^2} = \sqrt{82\,100}$$
$$c \doteq 286.531$$
$$c \doteq 287$$

1. Find c in ΔABC.



2. Find c in ΔABC.



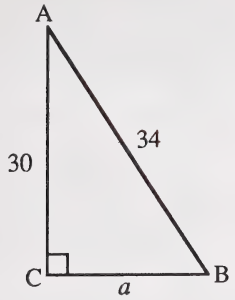


ABCD is a rectangle. $\overline{AC}$ and $\overline{BD}$ are diagonals.

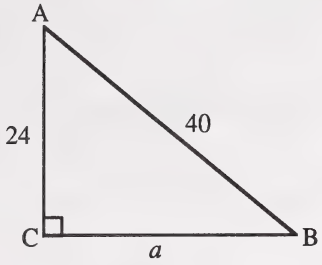
Start the solution with a simple diagram.

Conclude the solution by using a statement.

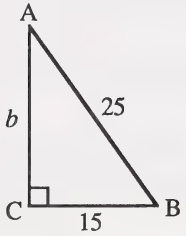
3. Find a in ΔABC.



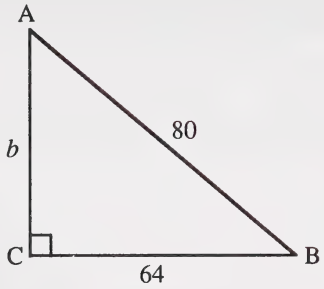
4. Find a in ΔABC.



5. Find b in ΔABC.

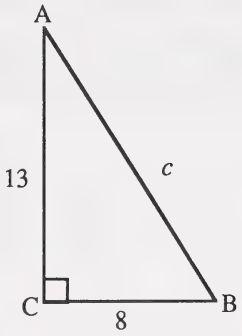


6. Find b in ΔABC.

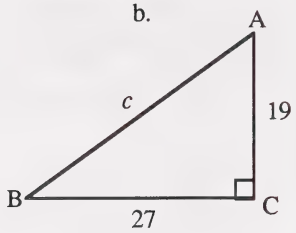


7. Do either part a or b. Solve for c and round the answer to the nearest whole number .

a.

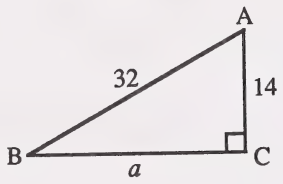


b.

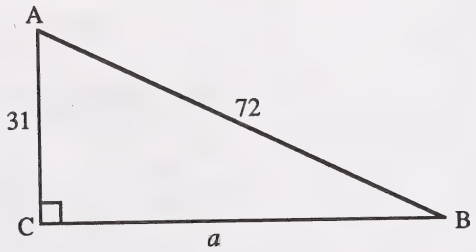


8. Do either part a or b. Solve for a and round the answer to the nearest tenth.

a.

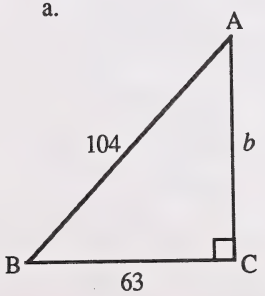


b.

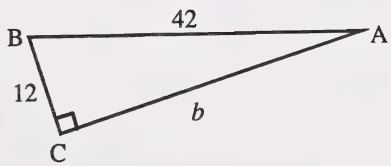


9. Do either part a or b. Solve for b and round the answer to the nearest hundredth.

a.



b.



Do all the remaining questions. Start each solution by making a sketch. Round each answer as indicated. Use a closing statement to complete each solution.

10. The foot of a 4.3 m ladder is placed 1.2 m away from the wall. How high up the wall does the ladder reach? Round the answer to the nearest tenth of a metre.

11. The back of a truck is 0.75 m above the ground. Find the length of a ramp needed to reach from the back of the truck to a point 3.15 m away. Round the answer to the nearest hundredth.

12. A hydrotower has an anchor 22 m from its base. The tower is anchored by a guy wire which is 43 m long. The shadow and the wire extend in the same direction. Find the height of the hydrotower. Round the answer to the nearest metre.

For solutions to **Activity 4**, turn to **Appendix A, Topic 1**.

If you require help, do the Extra Help section.

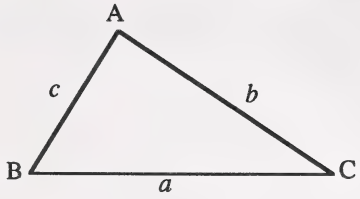
If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

Keep the following points in mind.

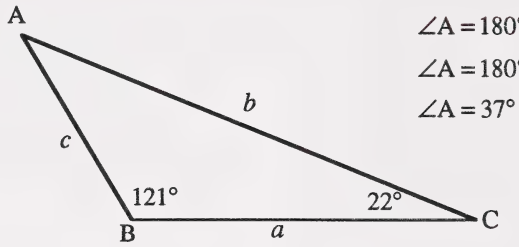
- Upper case letters are used to name the vertices of a triangle.
- Lower case letters are used to name the sides of a triangle.
- The corresponding letters for the vertices and the sides are always opposite each other.

The following is an example of the points mentioned above.



- The sum of the three angles of any triangle is 180°. To find a third angle, add the measure of the two given angles and subtract this sum from 180°.

For example, find the measure of $\angle A$ in ΔABC.

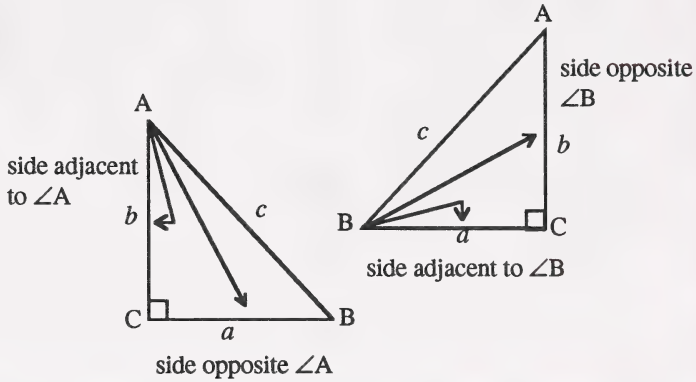


$\angle A = 180° - (121° + 22°)$

$\angle A = 180° - 143°$

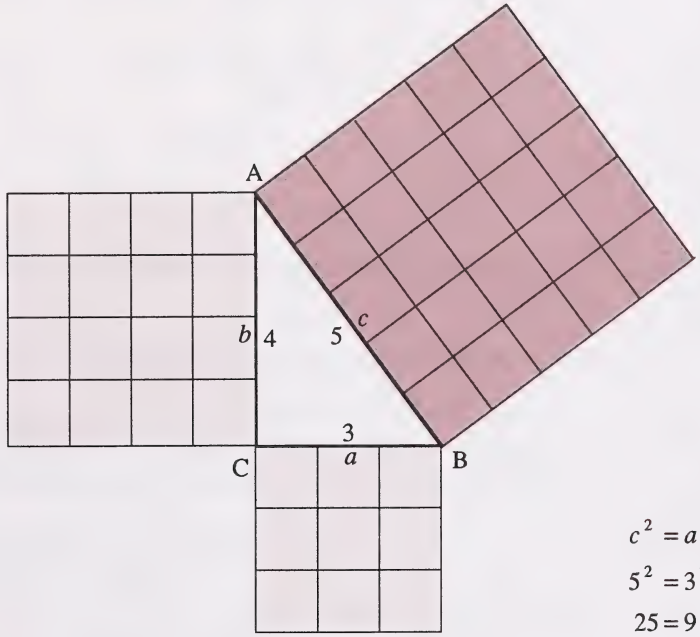
$\angle A = 37°$

- A diagram can be used to illustrate the side opposite a given angle and the sides adjacent to a given angle.



- The Theorem of Pythagoras relationship can be shown in diagram form when the following values are used in ΔABC.

$a = 3$
$b = 4$
$c = 5$



$$c^2 = a^2 + b^2$$
$$5^2 = 3^2 + 4^2$$
$$25 = 9 + 16$$
$$25 = 25$$

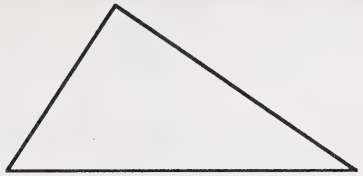
For further application of these basic concepts, do the questions which follow.

Adaptions:

- $a^2 = c^2 - b^2$
  $3^2 = 5^2 - 4^2$
  $9 = 25 - 16$
  $9 = 9$

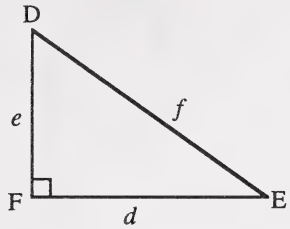
- $b^2 = c^2 - a^2$
  $4^2 = 5^2 - 3^2$
  $16 = 25 - 9$
  $16 = 16$

1. Use L, M, and N to label the given triangle.



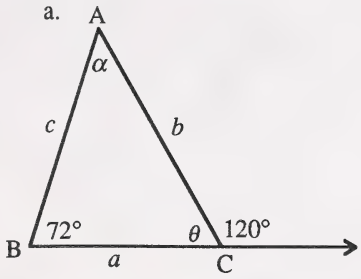
2. a. What are the sides opposite to the two acute angles and the sides adjacent to the two acute angles in right ΔDEF?

   b. Identify the hypotenuse.

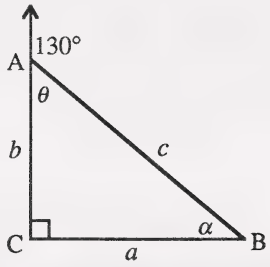


3. Find the measures of the angles indicated in each diagram.

a.



b.

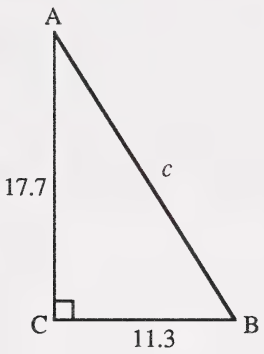


4. Find the measure for each unknown side.

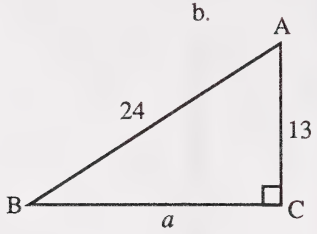
   Round the answers to

   i. the nearest whole number
   ii. the nearest tenth
   iii. the nearest hundredth
   iv. the nearest thousandth

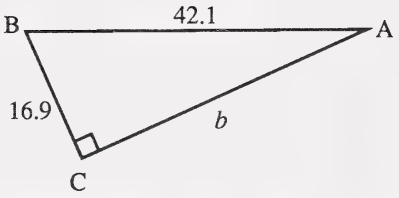
a.



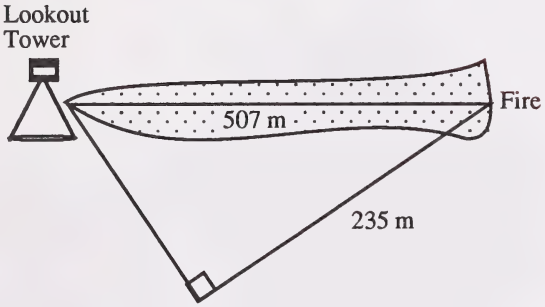
b.



c.



5. A forest ranger spotted some smoke which was at the other end of a lake. He knew this distance to be 507 m. If one side of an imagined right triangle illustrated below is 235 m, find the length of the other side to the nearest metre.



For solutions to **Extra Help**, turn to **Appendix A, Topic 1.**

Extensions

The measures of the three angles of a triangle can be represented by algebraic expressions. For example, in ΔABC, $\angle A = (2x + 1)°$, $\angle B = (3x - 4)°$, and $\angle C = (3x - 1)°$.

To find the exact measure of each angle in ΔABC, proceed as shown here. Use your equation-solving skills to find x.

$$\angle A + \angle B + \angle C = 180°$$
$$(2x+1)+(3x-4)+(3x-1)=180$$
$$8x-4=180$$
$$8x=180+4$$
$$8x=184$$
$$\frac{8x}{8}=\frac{184}{8}$$
$$x=23$$

To find the measure of each angle, substitute $x = 23$ in the original expressions.

Therefore,

$$\angle A = 2x+1$$
$$=2(23)+1$$
$$=46+1$$
$$=47°$$

$$\angle B = 3x-4$$
$$=3(23)-4$$
$$=69-4$$
$$=65°$$

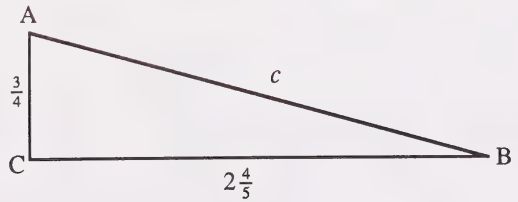
$$\angle C = 3x-1$$
$$=3(23)-1$$
$$=69-1$$
$$=68°$$

Total = 47° + 65° + 68° = 180°

In addition to using whole numbers and decimal numbers as measures of sides of a right triangle, other types of values such as fractions, mixed numbers, and radicals often replace the originally mentioned ones. For the fractions and mixed numbers, rounding is often required. When radicals are used, the answers should be simplified.

Example 8

Find c.



Solution:

$$c^2 = a^2 + b^2$$

$$c^2 = \left(2\tfrac{4}{5}\right)^2 + \left(\frac{3}{4}\right)^2 \quad \text{or} \quad c^2 = (2.8)^2 + (0.75)^2$$

$$c^2 = \left(\frac{14}{5}\right)^2 + \left(\frac{3}{4}\right)^2 \qquad c^2 = 7.84 + 0.5625$$

$$c^2 = \frac{196}{25} + \frac{9}{16} \qquad c^2 = 8.4025$$

$$c^2 = \frac{3136 + 225}{400} \qquad c \doteq 2.899$$

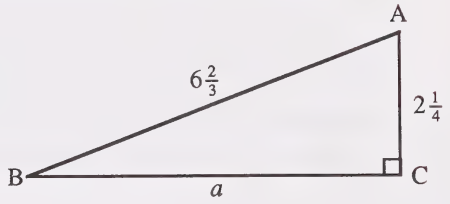
$$c = \sqrt{\frac{3361}{400}}$$

$$c = \sqrt{8.4025}$$

$$\doteq 2.899$$

Example 9

Solve for a. Round the values to two decimal places.



Solution:

$$a^2 = c^2 - b^2$$

$$a^2 = \left(6\tfrac{2}{3}\right)^2 - \left(2\tfrac{1}{4}\right)^2 \quad \text{or} \quad a^2 = (67)^2 - (2.25)^2$$

$$a^2 = \left(\frac{20}{3}\right)^2 - \left(\frac{9}{4}\right)^2 \qquad a^2 = 44.4889 - 5.0625$$

$$a^2 = \frac{400}{9} - \frac{81}{16} \qquad a^2 = 39.4264$$

$$a^2 = \frac{6400 - 729}{144} \qquad a = 6.28$$

$$a^2 = \frac{5671}{144}$$

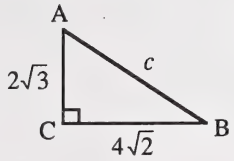
$$\sqrt{a^2} = \sqrt{\frac{5671}{144}}$$

$$a \doteq \sqrt{39.3819}$$

$$a \doteq 6.28$$

Example 10

Solve for c. Leave the answer in a simplified radical form and in evaluated form rounded to three decimal places.



Solution:

$$c^2 = a^2 + b^2$$
$$c^2 = \left(4\sqrt{2}\right)^2 + \left(2\sqrt{3}\right)^2$$
$$c^2 = 32 + 12$$
$$c^2 = 44$$
$$\sqrt{c^2} = \sqrt{44}$$
$$c = 2\sqrt{11} \text{ or } c \doteq 6.633$$

As a radical, $c = 2\sqrt{11}$.

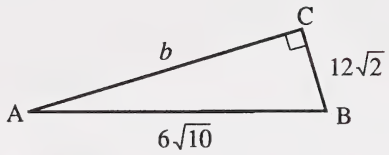
As a decimal number, $c \doteq 6.633$.

Note: The radical value of $2\sqrt{11}$ for c is exact.

The radical answer is exact while the decimal answer is approximate.

Example 11

Solve for b. Give the answer as a simplified radical and a decimal number rounded to two decimal places.



Solution:

$$c^2 = a^2 + b^2$$
$$b^2 = c^2 - a^2$$
$$b^2 = \left(6\sqrt{10}\right)^2 - \left(12\sqrt{2}\right)^2$$
$$b^2 = 360 - 288$$
$$b^2 = 72$$
$$\sqrt{b^2} = \sqrt{72}$$
$$b = 6\sqrt{2}$$

The radical value is $6\sqrt{2}$.

The decimal value is $6\sqrt{2} \doteq 6 \times 1.414$
$\doteq 8.485$
$\doteq 8.49$.

Note: $\doteq$ means approximately equal to.

$$\left(6\sqrt{10}\right)^2 = \left(6\sqrt{10}\right)\left(6\sqrt{10}\right)$$
$$= 36 \times 10$$
$$= 360$$

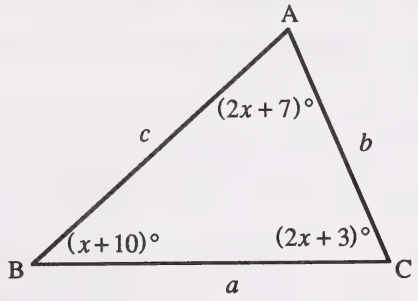
$$\left(12\sqrt{2}\right)^2 = \left(12\sqrt{2}\right)\left(12\sqrt{2}\right)$$
$$= 144 \times 2$$
$$= 288$$

$$\left(4\sqrt{2}\right)^2 = \left(4\sqrt{2}\right)\left(4\sqrt{2}\right)$$
$$= 16 \times 2$$
$$= 32$$

$$\left(2\sqrt{3}\right)^2 = \left(2\sqrt{3}\right)\left(2\sqrt{3}\right)$$
$$= 4 \times 3$$
$$= 12$$

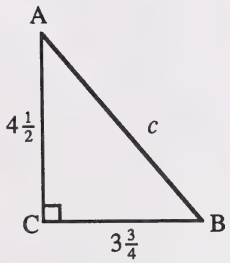
Try the following questions. Remember to use a calculator whenever possible.

1. Find the measures of the three angles of ΔABC.

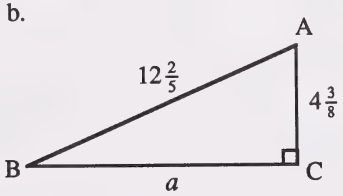


2. Solve for the unknown side in each triangle. Approximate the answers to two decimal places.

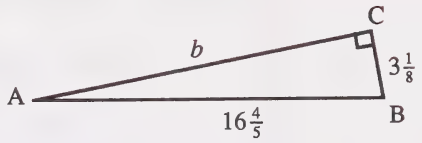
a.



b.

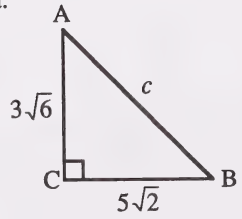


c.

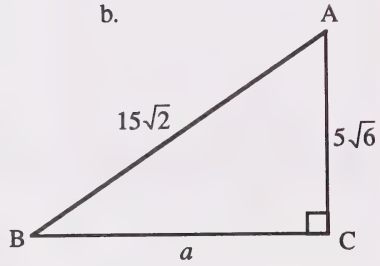


3. Solve for the unknown side in each triangle. Express each answer as a simplified radical and as a decimal number to two places.

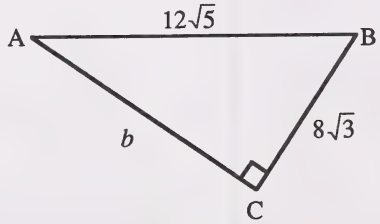
a.



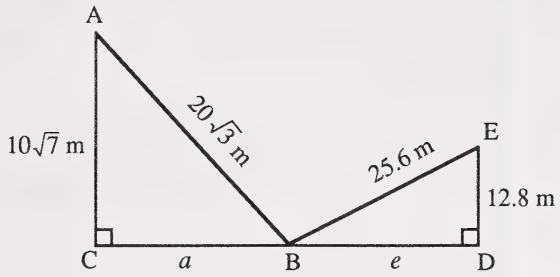
b.



c.



4. In the diagram provided, find the distance between point C and point D. Round all numbers to two decimal places.



For solutions to **Extensions**, turn to **Appendix A, Topic 1**.

Topic 2 Similar Triangles

Introduction

In your travels, have you ever stopped to wonder how the heights of mountains are found? No doubt you have seen signs indicating how high a mountain reaches. Do you think that an actual metre stick was used to find such measurements? Do you think that Mount Kilimanjaro, which is 5895 m high and is the highest mountain in Africa, was actually measured using a metre stick? The answer is no. This topic should help you understand what is known as indirect measurement using similar triangles.

What Lies Ahead

Throughout the topic you will learn to

1. define similar triangles and identify their properties
2. use the ratio property of similar triangles to find the lengths of corresponding sides

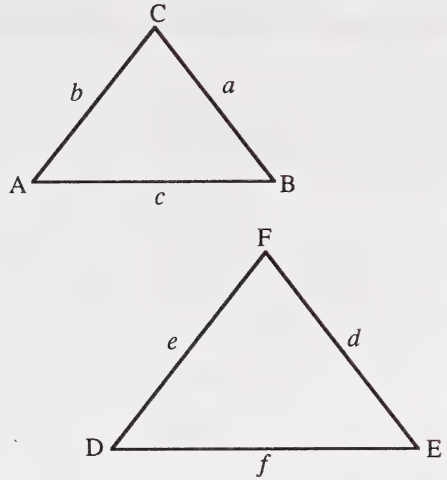
Now that you know what to expect, turn the page to begin your study of similar triangles.

Exploring Topic 2

Activity 1

Define similar triangles and identify their properties.

Take a moment to examine your immediate surroundings. Without too much effort, you will notice many shapes and forms which seem to be similar. All circles are similar. Other geometric figures have many similar features, even though their sizes may differ. All squares have four equal sides and four angles which measure 90° each. All rectangles have opposite sides that are equal and four angles which measure 90° each. These are just a few of many examples. Your main concern will be working with triangles which are similar in nature. Study these two triangles.



ΔABC and ΔDEF are similar because they have the same shape. The size is different, but this is of no consequence.

In similar triangles, corresponding angles are equal and the ratios of corresponding sides are equal.

For the above triangles,

$\angle A \leftrightarrow \angle D$, because $\angle A = \angle D$

$\angle C \leftrightarrow \angle F$, because $\angle C = \angle F$

$\angle B \leftrightarrow \angle E$, because $\angle B = \angle E$

A triangle is a polygon which has three sides.

Triangles which have the same shape and the same size are called congruent triangles.

$\angle A \leftrightarrow \angle D$ is read $\angle A$ corresponds to $\angle D$. Similarly, $\angle C \leftrightarrow \angle F$ means $\angle C$ corresponds to $\angle F$, and $\angle B \leftrightarrow \angle E$ means that $\angle B$ corresponds to $\angle E$.

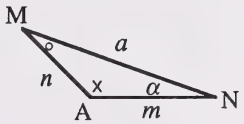
side $a \leftrightarrow$ side d, because a is opposite $\angle A$ and d is opposite $\angle D$.

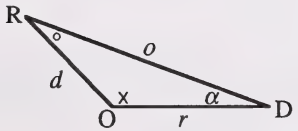
side $c \leftrightarrow$ side f, because c is opposite $\angle C$ and f is opposite $\angle F$.

side $b \leftrightarrow$ side e, because b is opposite $\angle B$ and e is opposite $\angle E$.

Because of the above properties,
$\Delta ABC \sim \Delta DEF$.

The way in which a similarity relationship is written can be used to name corresponding angles and corresponding sides. Take note of ΔMAN and ΔROD which follow.





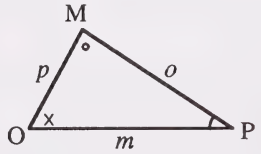
If $\Delta MAN \sim \Delta ROD$,
$\angle M \leftrightarrow \angle R$ or $\angle AMN \leftrightarrow \angle ORD$.
Also, $\overline{MA} \leftrightarrow \overline{RO}$ or $n \leftrightarrow d$.

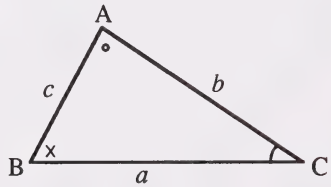
If $\Delta MAN \sim \Delta ROD$,
$\angle A \leftrightarrow \angle O$ or $\angle MAN \leftrightarrow \angle ROD$.
Also, $\overline{MN} \leftrightarrow \overline{RD}$ or $a \leftrightarrow o$.

If $\Delta MAN \sim \Delta ROD$,
$\angle N \leftrightarrow \angle D$ or $\angle ANM \leftrightarrow \angle ODR$.
Also, $\overline{AN} \leftrightarrow \overline{OD}$ or $m \leftrightarrow r$.

Do either the odd- or even-numbered questions.

1. List all the similarity relationships for ΔABC and ΔMOP in two ways.





The equal ratios for the corresponding sides in ΔABC and ΔDEF are

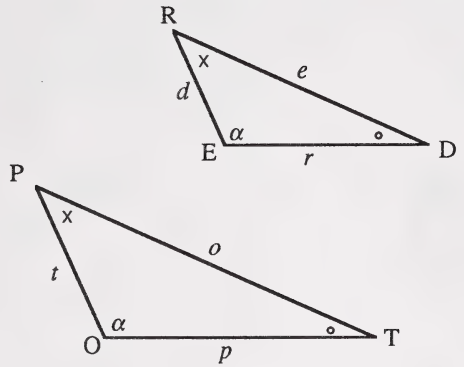
$$\frac{a}{d} = \frac{c}{f} = \frac{b}{e}.$$

The symbol ~ means similar to.

Note the various symbols used to show which angles in ΔMAN and ΔROD are equal.

The sides of a Δ are line segments. $\overline{MA}$ is read as segment MA. Similarly, $\overline{RO}$, $\overline{MN}$, $\overline{RD}$, $\overline{AN}$, and $\overline{OD}$ are segments RO, MN, RD, AN, and OD.

2. List all the similarity relationships for ΔRED and ΔPOT in two ways.



3. List all the similarity relationships for ΔCAT ~ ΔPON using single letters for the angles and line segment notation for the sides.

4. List all the similarity relationships for ΔWXY ~ ΔDOT using single letters for the angles and line segment notation for the sides.

For solutions to **Activity 1**, turn to **Appendix A, Topic 2.**

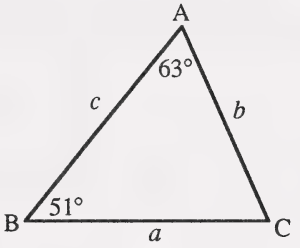
Activity 2

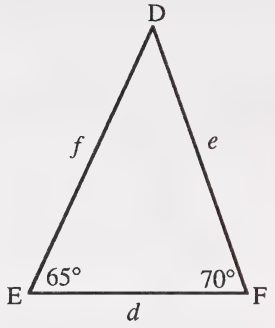
Use the ratio property of similar triangles to find the lengths of corresponding sides.

If two angles of one triangle are equal to two angles of another triangle, the triangles are similar. To show that the third angles are also equal, use the fact that the sum of the three angles of any triangle is 180°.

Example 1

Find the measure of the third angle in ΔABC and ΔDEF.





Solution:

$\angle C = 180° - (63° + 51°)$

$\angle C = 180° - 114°$

$\angle C = 66°$

$\angle D = 180° - (65° + 70°)$

$\angle D = 180° - 135°$

$\angle D = 45°$

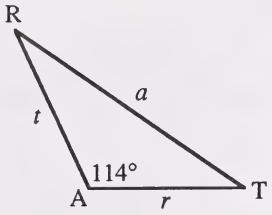
Try the following exercises.

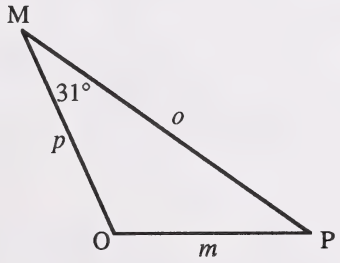
Complete at least one of the following questions.

1. Assume that the triangles are similar.

   a. Find the measure of each of the unknown angles in each triangle.

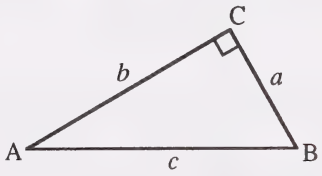
   b. List all the similarity relationships for the resulting similar triangles.

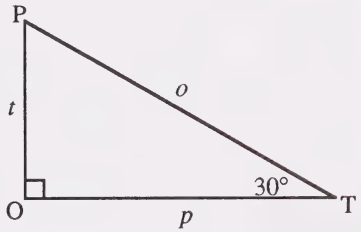




2. a. Find the measure of each unknown angle in each triangle.

   b. List all the similarity relationships for the resulting similar triangles.

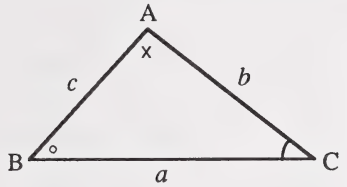


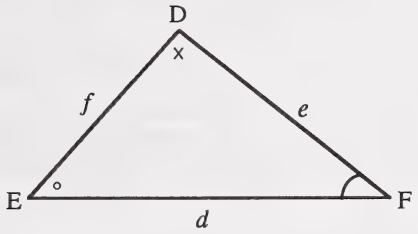


For solutions to **Activity 2**, turn to **Appendix A, Topic 2**.

Corresponding angles of similar triangles are equal.

It was mentioned earlier that similar triangles have two properties. The first property is that corresponding angles are equal. The second property states that corresponding sides are proportional or that the ratios of corresponding sides are equal. This means that if $\Delta ABC \sim \Delta DEF$, the ratios of the corresponding sides are equal.





From ΔABC to ΔDEF,

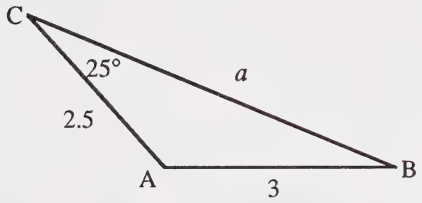
$a \leftrightarrow d$
$b \leftrightarrow e$
$c \leftrightarrow f$

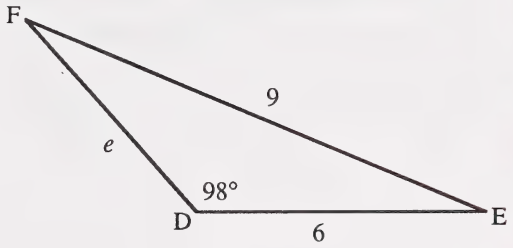
Therefore, $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$.

Using these two properties of similar triangles, you can find all unknown measures whether they are measures of angles or measures of sides.

Example 2

Given that $\Delta ABC \sim \Delta DEF$, find all the unknown measures.





Solution:

The unknown measures are $\angle A$, $\angle B$, $\angle E$, $\angle F$, side a, and side e.

Since $\Delta ABC \sim \Delta DEF$,
$\angle A = \angle D$, so $\angle A = 98°$,
and $\angle C = \angle F$, so $\angle F = 25°$.

These ratios which result in proportions can be expressed in several different ways.

- $\frac{a}{b} = \frac{d}{e}$ or $\frac{b}{a} = \frac{e}{d}$
- $\frac{a}{c} = \frac{d}{f}$ or $\frac{c}{a} = \frac{f}{d}$
- $\frac{b}{c} = \frac{e}{f}$ or $\frac{c}{b} = \frac{f}{e}$

To find $\angle B$, use the fact that the sum of the three angles of any triangle is 180°.

$$\angle A + \angle B + \angle C = 180°$$
$$98° + \angle B + 25° = 180°$$
$$\angle B + 123° = 180°$$
$$\angle B = 180° - 123°$$
$$\angle B = 57°$$

Since $\angle B = \angle E$, then $\angle E = 57°$.

To find the measures of sides a and e, use the property that the corresponding sides of similar triangles are proportional.

That is, $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$.

Substitute the known values into these ratios.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$
$$\frac{a}{9} = \frac{2.5}{e} = \frac{3}{6}$$

By using the first and the last ratios, you can solve for a.

$$\frac{a}{9} = \frac{3}{6}$$
$$6a = 27$$
$$\frac{6a}{6} = \frac{27}{6}$$
$$a = 4.5$$

By using the middle and last ratios, you can solve for e.

$$\frac{2.5}{e} = \frac{3}{6}$$
$$3e = 15$$
$$\frac{3e}{3} = \frac{15}{3}$$
$$e = 5$$

The following is a summary of the angles and measures.

$\angle A = 98°$ side $a = 4.5$

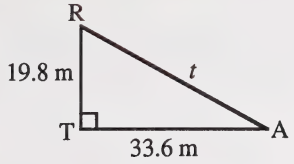
$\angle B = 57°$ side $e = 5$

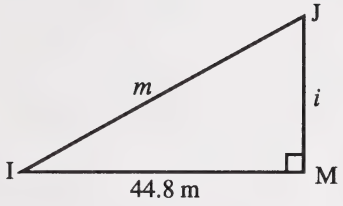
$\angle E = 57°$

$\angle F = 25°$

Choose the ratios which give a proportion in which three of the four values are known.

Example 3





- In ΔRAT and ΔJIM, ∠A = ∠I and ∠T = ∠M = 90°. Why is ΔRAT ~ ΔJIM?

Solution:

Since ∠A = ∠I and ∠T = ∠M, and the sum of the angles in a triangle is 180°, ∠R = ∠J.

Thus, ΔRAT ~ ΔJIM, since all corresponding angles are equal.

- Use the given measures to find the length of $\overline{JM}$.

Solution:

Since ΔRAT ~ ΔJIM, corresponding sides are proportional.

$$\frac{r}{j} = \frac{t}{m} = \frac{a}{i}$$

or $$\frac{33.6}{44.8} = \frac{t}{m} = \frac{19.8}{i}$$

Use the first and the last ratios to solve for i.

$$\frac{33.6}{44.8} = \frac{19.8}{i}$$

$$33.6i = 19.8 \times 44.8$$

$$i = \frac{19.8 \times 44.8}{33.6}$$

$$i = \frac{887.04}{33.6}$$

$$i = 26.4$$

The length of $\overline{JM}$ is 26.4 m.

This third example proves this fact:

If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.

$\overline{JM}$ is also side i in ΔJIM.

When corresponding sides are proportional, the ratios for these sides are equal.

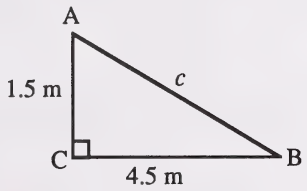
The property of corresponding sides being proportional in similar triangles can be used to solve practical problems. Look at the example which follows.

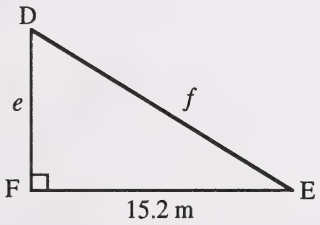
Example 4

A tree casts a shadow 15.2 m in length at the same time that a fence post which is 1.5 m high casts a shadow 4.5 m in length. Find the height of the tree to one decimal place.

Solution:

Sketch a diagram and record the given information.





Let e be the height of the tree.

Since $\Delta ABC \sim \Delta DEF$, the use of equal ratios is possible.

Choose and use the two ratios where three of the four values are known.

$$\frac{b}{e} = \frac{a}{d}$$

$$\frac{1.5}{e} = \frac{4.5}{15.2}$$

$$4.5e = 1.5 \times 15.2$$

$$4.5e = 22.8$$

$$\frac{4.5e}{4.5} = \frac{22.8}{4.5}$$

$$e \doteq 5.067$$

$$e \doteq 5.1$$

The height of the tree is approximately 5.1 m.

The above solution could have been done using a different pair of ratios. Compare two sides of one triangle for the first ratio and two sides of the second triangle for the second ratio.

Substitute the known values in the resulting proportion.

Be sure that the order of comparison is the same for both ratios. The use of $\frac{b}{a} = \frac{d}{e}$ would not work, since the order of comparison is not the same in both ratios.

$$\frac{b}{a}=\frac{e}{d}$$

$$\frac{1.5}{4.5}=\frac{e}{15.2}$$

$$4.5e=1.5\times 15.2$$

$$4.5e=22.8$$

$$\frac{4.5e}{4.5}=\frac{22.8}{4.5}$$

$$e\doteq 5.067$$

$$e\doteq 5.1$$

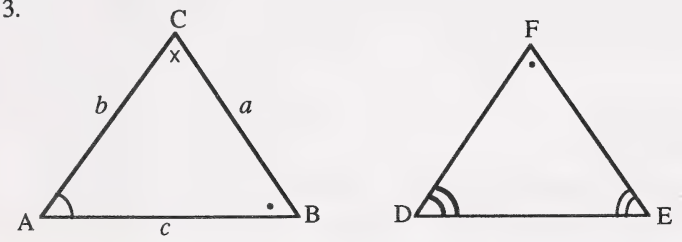
The height of the tree is approximately 5.1 m.

The second solution shows that there are always two ways that can be used to set up a proportion when finding the length for a side of any one of the two similar triangles.

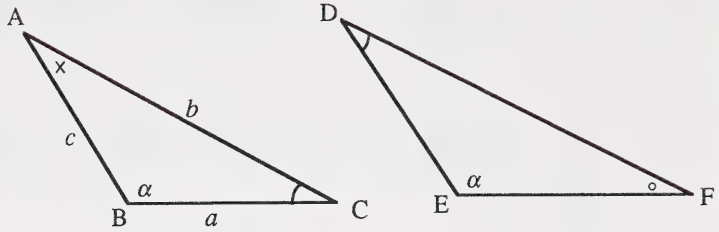
Do either question 3 or 4 and either question 5 or 6.

a. For each pair of triangles, state whether or not the triangles are similar.
b. If they are similar, list the corresponding angles and the corresponding ratios.

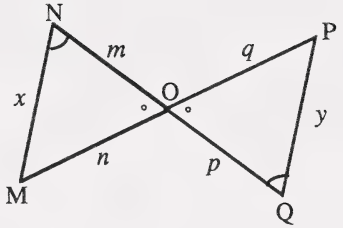
3.



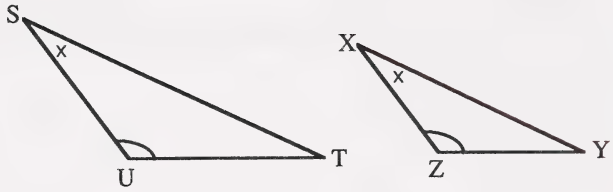
4.



5.



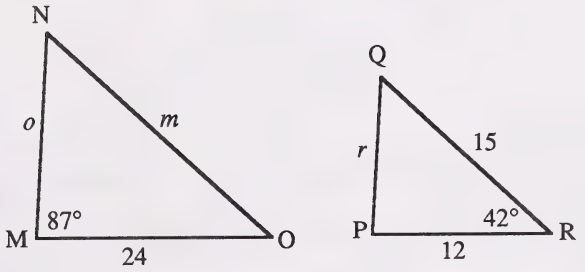
6.



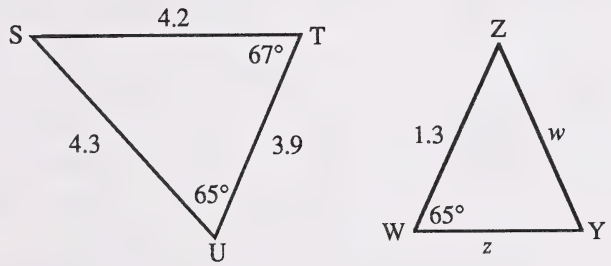
Do at least two parts of question 7.

7. Given the following pairs of similar triangles, find the unknown measures. For the side measures, round to one decimal place if necessary.

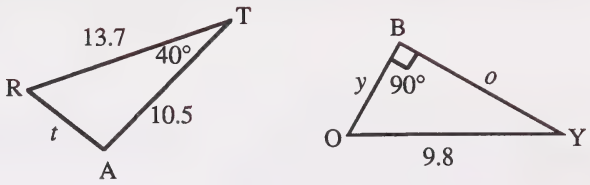
a.



b.

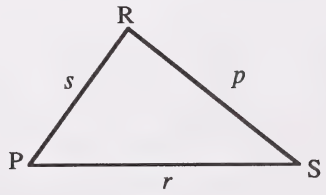


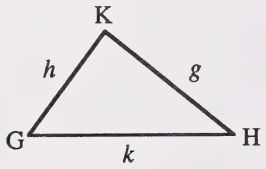
c.



Now complete the following questions.

8. Given $\Delta PSR \sim \Delta GHK$, complete the following proportions.





a. $\frac{p}{g} = \frac{r}{\boxed{?}}$

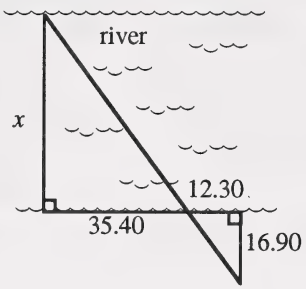
b. $\frac{s}{\boxed{?}} = \frac{h}{g}$

c. $\frac{r}{s} = \frac{\boxed{?}}{h}$

d. $\frac{\boxed{?}}{s} = \frac{g}{p}$

9. The shadow, $\overline{BC}$, of a 7.3 m vertical pole is 11.8 m long, and the shadow, $\overline{XY}$, of a shed is 16.4 m long. Assuming that $\Delta ABC \sim \Delta ZYX$, what is the height, $\overline{ZY}$, of the shed? Round the answer to the nearest tenth of one metre.

10. In the diagram, the distance across the river is represented by x. Using the given measurements, find the width of the river to the nearest hundredth of a metre.



For solutions to **Activity 2**, turn to **Appendix A**, **Topic 2**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

Two triangles are similar when the following properties or conditions exist.

- The two triangles have the same shape, but not the same size.
- Corresponding angles are equal. The symbol $\leftrightarrow$ means corresponds to.
- Corresponding sides are proportional. This means that the ratios of the corresponding sides are equal.

  Ratio example: $\frac{x}{y}$

  Proportion example: $\frac{a}{b}=\frac{c}{d}$
- When two angles of one triangle are equal to two angles of a second triangle, the triangles are similar. This is true because the sum of the three angles of any triangle is 180°.
- If $\Delta ABC \sim \Delta DEF$, the similarity relationships can be determined without a diagram. To check, a diagram like the one that follows can be used.

Similarity relationships from $\Delta ABC \sim \Delta DEF$:

From $\Delta ABC \sim \Delta DEF$,

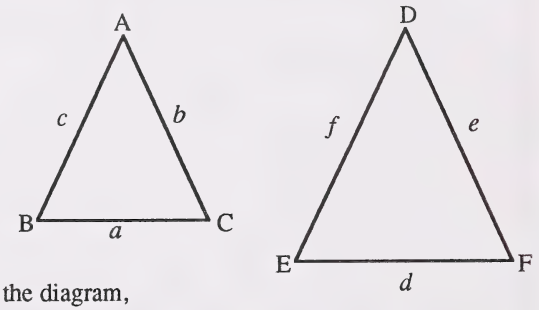
$\angle A \leftrightarrow \angle D$ and $\overline{AB} \leftrightarrow \overline{DE}$

$\angle B \leftrightarrow \angle E$ and $\overline{BC} \leftrightarrow \overline{EF}$

$\angle C \leftrightarrow \angle F$ and $\overline{AC} \leftrightarrow \overline{DF}$

Hint: $\overline{AB} \leftrightarrow \overline{DE}$ is read as segment AB corresponds to segment DE.

Similarity relationships from a diagram:



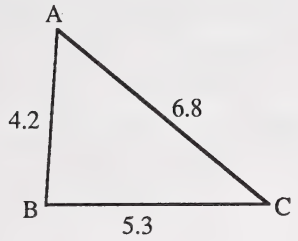
From the diagram,

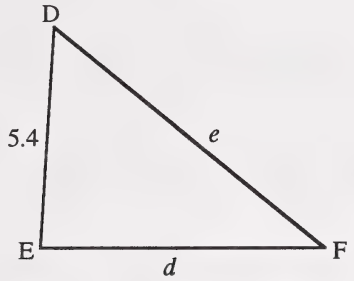
$\angle A \leftrightarrow \angle D$ and $\overline{AB} \leftrightarrow \overline{DE}$ or $c \leftrightarrow f$

$\angle B \leftrightarrow \angle E$ and $\overline{BC} \leftrightarrow \overline{EF}$ or $a \leftrightarrow d$

$\angle C \leftrightarrow \angle F$ and $\overline{AC} \leftrightarrow \overline{DF}$ or $b \leftrightarrow e$

- For similar triangles, the measure of an unknown side can be calculated by setting up a proportion using two ratios. Choose a proportion where three of the four values are known. Study this situation.





$\Delta ABC \sim \Delta DEF$

From the diagrams, you can see $\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$.

Therefore, $\frac{5.3}{d}=\frac{6.8}{e}=\frac{4.2}{5.4}$.

To find d, use $\frac{5.3}{d}=\frac{4.2}{5.4}$.

$4.2d = 5.3 \times 5.4$

$4.2d = 28.62$

$d \doteq 6.814$

$d \doteq 6.8$ (to the nearest tenth)

To find e, use $\frac{6.8}{e}=\frac{4.2}{5.4}$.

$4.2e = 6.8 \times 5.4$

$4.2e = 36.72$

$e \doteq 8.743$

$e \doteq 8.7$ (to the nearest tenth)

The values for d and e were rounded to the nearest tenth because the given measures are in tenths.

- For the previous triangles in number 6, other ratios could have been used to solve for d and e. Take note of how this can be done.

To solve for d, you use the ratio $\frac{c}{a}=\frac{f}{d}$.

$\frac{c}{a}=\frac{f}{d}$

$\frac{4.2}{5.3}=\frac{5.4}{d}$

$4.2d = 5.4 \times 5.3$

$4.2d = 28.62$

$d \doteq 6.814$

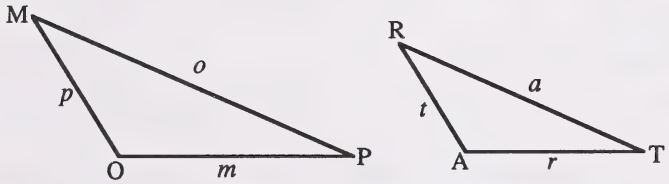
$d \doteq 6.8$ (to the nearest tenth)

To solve for e, you use the ratio $\frac{c}{b}=\frac{f}{e}$.

$$\frac{c}{b}=\frac{f}{e}$$
$$\frac{4.2}{6.8}=\frac{5.4}{e}$$
$$4.2e=6.8\times 5.4$$
$$4.2e=36.72$$
$$e\doteq 8.743$$
$$e\doteq 8.7 \qquad \text{(to the nearest tenth)}$$

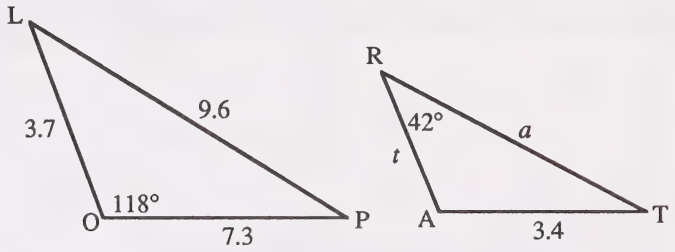
Try to keep all of these points in mind as you do the following exercises.

1. If $\Delta MOP \sim \Delta RAT$, give all the similarity relationships.



2. Without a diagram, list the similarity relationships if $\Delta POT \sim \Delta MAN$.

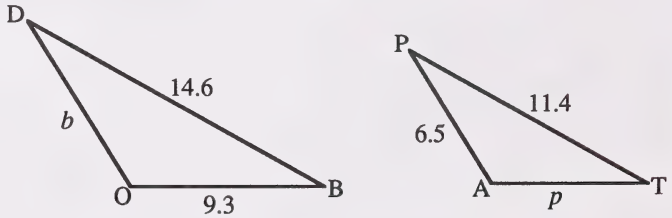
3. Find the missing measures if $\Delta LOP \sim \Delta RAT$. Express the side measures to the nearest tenth.



4. Given that $\Delta DOB \sim \Delta PAT$, do the following.

   a. Solve for the unknown side measures using the appropriate proportion from $\frac{b}{t}=\frac{d}{p}=\frac{o}{a}$.

   b. Check your solutions by using the appropriate proportion from the two provided.



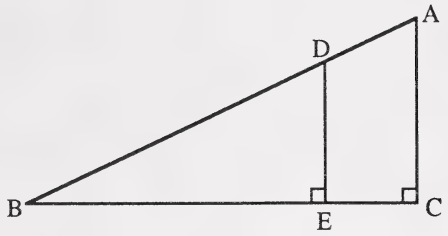
Express all measures to the nearest tenth.

For solutions to **Extra Help**, turn to **Appendix A, Topic 2.**

Extensions

As you do the exercises in this section, keep in mind these facts.

- If $\Delta ABC \sim \Delta DEF$, corresponding angles are equal and the ratios of corresponding sides are equal.
- When working with similar triangles which overlap, use two separate triangles to identify the similarity correspondences. For example, in this sketch



ΔABC and ΔDBE overlap each other. Separate the two to see that the following is true:

$\angle B \leftrightarrow \angle B$ and $\overline{BD} \leftrightarrow \overline{BA}$

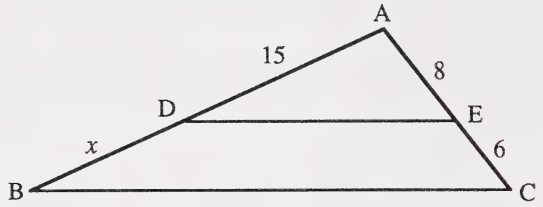
$\angle D \leftrightarrow \angle A$ and $\overline{BE} \leftrightarrow \overline{BC}$

$\angle E \leftrightarrow \angle C$ and $\overline{DE} \leftrightarrow \overline{AC}$

- Sometimes, unknowns are used for portions of the corresponding sides in similar triangles. Study the example which follows to see how the measure of such unknown sides can be found or portions thereof can be found.

Example 5

Given that $\Delta ABC \sim \Delta ADE$, find the value of x.



Solution:

$$\frac{AB}{AD} = \frac{AC}{AE}$$

$$\frac{15+x}{15} = \frac{8+6}{8}$$

$$8(15+x) = 15 \times 14$$

$$120 + 8x = 210$$

$$8x = 90$$

$$x = 11.25$$

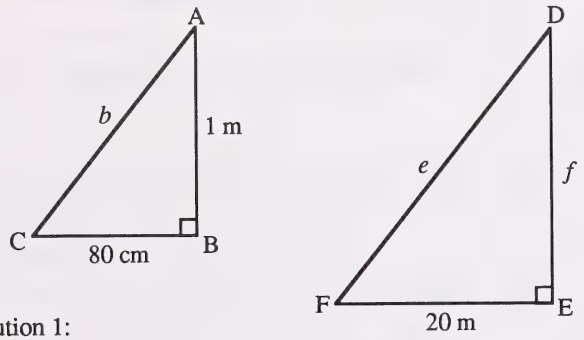
The value of x is 11.25 units.

- On other occasions, the given measures may not be expressed in the same units. When this happens, a change to the same units is required before solving for the unknown measures. This sort of situation is illustrated below.

Example 6

$\Delta ABC \sim \Delta DEF$

Find the measure of $\overline{DE}$ which is marked f.



Solution 1:

First change 80 cm to metres.

$$80 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0.8 \text{ m}$$

$$\frac{1}{f} = \frac{0.8}{20}$$

$$0.8f = 20$$

$$f = \frac{20}{0.8}$$

$$f = 25$$

The value for f or the measure of $\overline{DE}$ is 25 m.

Solution 2:

You could also change 1 m to 100 cm and 20 m to 2000 cm. The solution shows how this is done.

$$\frac{100}{f} = \frac{80}{2000}$$

$$80f = 200\,000$$

$$f = \frac{200\,000}{80}$$

$$f = 2500$$

The value for f or the measure of $\overline{DE}$ is 2500 cm.

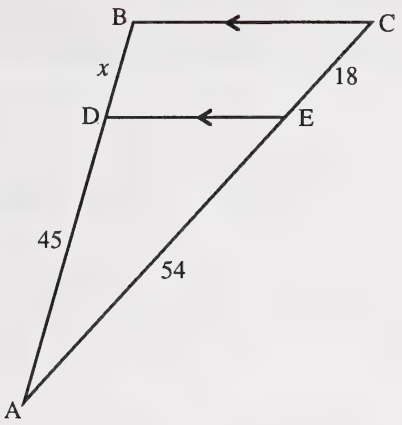
As you can see, both solutions have the same result.

$$2500 \text{ cm} = 2500 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}}$$

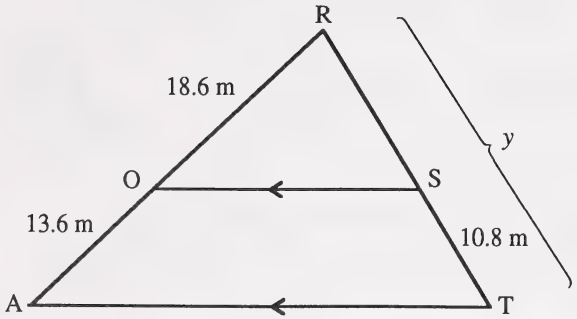
$$= 25 \text{ m}$$

Work through the following problems on your own.

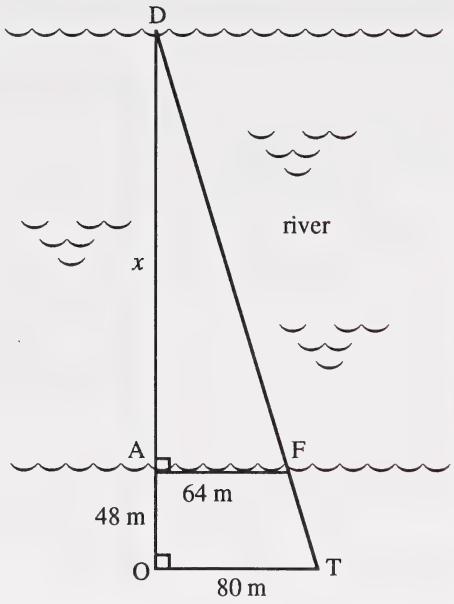
1. Solve for x.

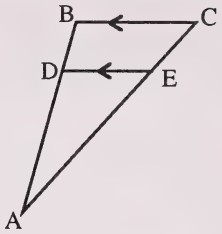


2. Solve for y. Round the answer to one decimal place.



3. Use the information in the sketch to find the distance, $\overline{DA}$, across the river.





The ← means that $\overline{BC}$ is parallel to $\overline{DE}$.

In brief, $\overline{BC} \parallel \overline{DE}$.

The symbol $\parallel$ means parallel to.

4. A vertical fence post 2 m in height casts a shadow of 150 cm. At the same time, a tree casts a shadow of 30 m. Find the height of the tree in metres and in centimetres.

5. A mirror is placed flat on the ground 80 m from the bottom of a tower. A person whose eye is 150 cm above the ground, moves until he or she can just see the top of the tower in the mirror. If the person is 320 cm from the mirror, find the height of the tower in metres and in centimetres.

For solutions to **Extensions**, turn to **Appendix A, Topic 2.**

Sketch diagrams for questions 4 and 5 to start the solutions.

Topic 3 Developing the Trigonometric Ratios

Introduction

The word **trigonometry** is made up of two Greek words – **trigonon** which means **triangle**, and **metria** which means **measurement**.

In its simplest sense, trigonometry means the measurement of triangles and the relations that exist between the sides, angles, and areas of triangles.

What Lies Ahead

Throughout this topic you will learn to

1. define the tangent, sine, and cosine ratios
2. determine the tangent, sine, and cosine ratios of any given acute angle
3. determine the measure of an acute angle given the tangent, sine, or cosine ratio
4. determine the tangent, sine, or cosine ratios given two sides of a right triangle

Now that you know what to expect, turn the page to begin your study of developing the trigonometric ratios.

Exploring Topic 3

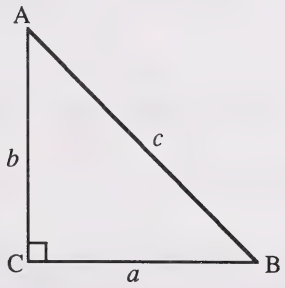
Activity 1

Define the tangent, sine, and cosine ratios.

Before the primary trigonometric ratios of an angle can be defined or developed, you must be able to select the side opposite a chosen angle and the side adjacent to this particular angle. You must also identify the hypotenuse.

Example 1

Name the side opposite and the side adjacent to both acute angles in right triangle ABC.



Solution:

The two acute angles are $\angle A$ and $\angle B$.

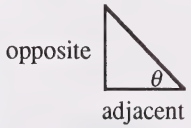
Angle:	Side opposite:	Side adjacent:
$\angle A$	$\overline{BC}$ or a	$\overline{AC}$ or b
$\angle B$	$\overline{AC}$ or b	$\overline{BC}$ or a

The hypotenuse is side $\overline{AB}$ or c.

Similar triangles can be used to develop and define trigonometric ratios.

To begin, concentrate your efforts on the tangent ratio. The tangent ratio for an angle is defined as the following:

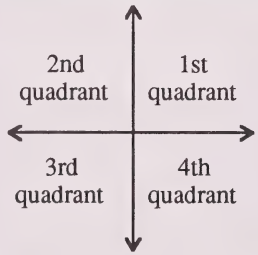
$$\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta}$$



Study these similar triangles which have been placed in the first quadrant of a coordinate plane. The angle chosen for any trigonometric ratio is called the reference angle.

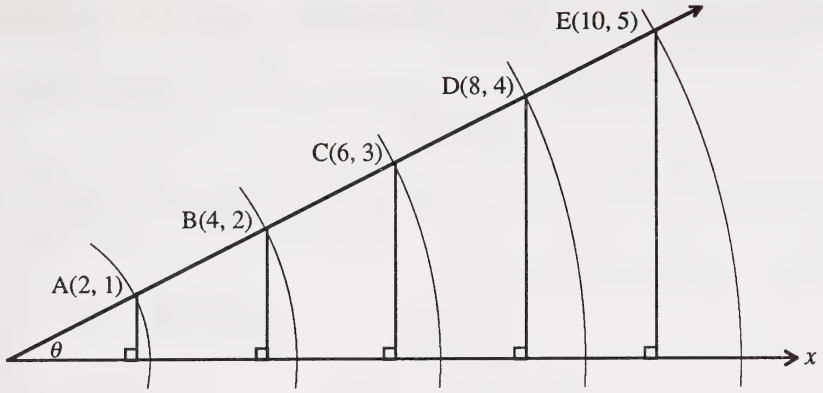
The hypotenuse is the longest side of a right triangle or the side opposite the right angle.

θ is the Greek letter theta which is commonly used to mark a reference angle.



The above is a coordinate plane with the quadrants named.

An acute angle is an angle which measures less than 90°.



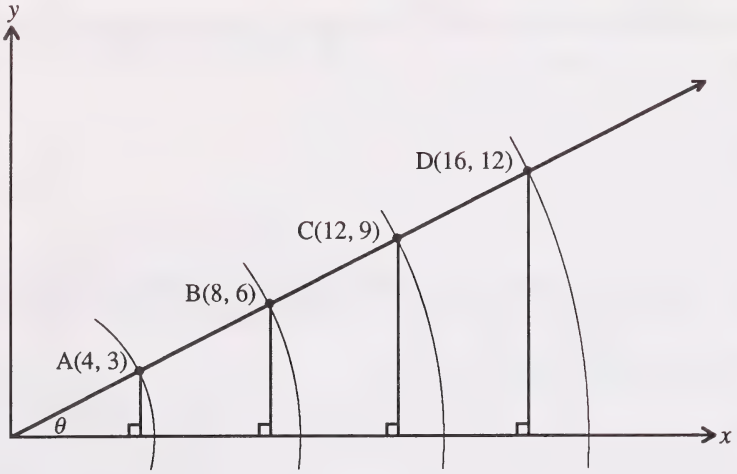
$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{1}{2};\ \frac{2}{4} = \frac{1}{2};\ \frac{3}{6} = \frac{1}{2};\ \frac{4}{8} = \frac{1}{2};\ \frac{5}{10} = \frac{1}{2}$$

Notice that the tan value for θ is always the same no matter how the size of the triangle changes. This means that the tan value for any particular angle is always the same.

Now you will continue with sine and cosine trigonometric ratios. Use the triangles placed in the first quadrant of a coordinate plane to define and develop these ratios.

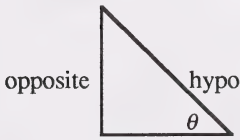
The angle θ always remains the same size in this diagram.

tan = tangent
sin = sine
cos = cosine



$$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{side adjacent}}{\text{hypotenuse}}$$



$$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{3}{5};\ \frac{6}{10} = \frac{3}{5};\ \frac{9}{15} = \frac{3}{5};\ \frac{12}{20} = \frac{3}{5}$$

The sine ratio is the same in all cases. This means that each acute angle has its own unique sine value which does not change.

for (4, 3)

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 4^2 + 3^2 \\ &= 16 + 9 \\ &= 25 \\ c &= 5 \end{aligned}$$

for (8, 6)

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 8^2 + 6^2 \\ &= 64 + 36 \\ &= 100 \\ c &= 10 \end{aligned}$$

for (12, 9)

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 12^2 + 9^2 \\ &= 144 + 81 \\ &= 225 \\ c &= 15 \end{aligned}$$

for (16, 12)

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 16^2 + 12^2 \\ &= 256 + 144 \\ &= 400 \\ c &= 20 \end{aligned}$$

$$\cos\theta = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$= \frac{4}{5};\ \frac{8}{10} = \frac{4}{5};\ \frac{12}{15} = \frac{4}{5};\ \frac{16}{20} = \frac{4}{5}$$

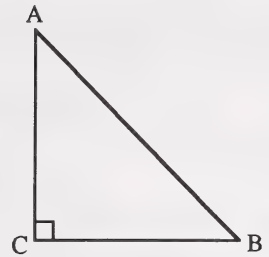
The cosine ratio is the same in all cases. This means that each acute angle has its own unique cosine value which is always the same.

It is time to do some exercises on your own.

Do either question 1 or 2.

1. Label the sides of each right triangle using lower case letters and determine the three primary ratios for each acute angle.

a.



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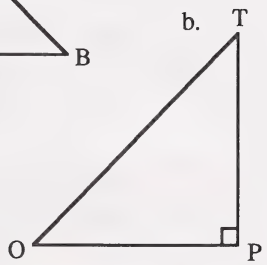
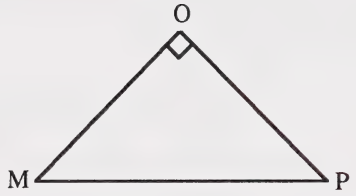


2. Label the sides of each right triangle using lower case letters and determine the three primary ratios for each acute angle.

a.



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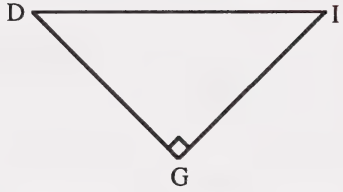


For solutions to **Activity 1**, turn to **Appendix A, Topic 3**.

The sine, cosine, and tangent ratios are often referred to as the primary trigonometric ratios of any acute angle.

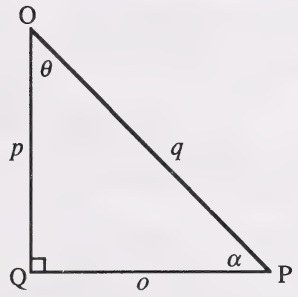
Activity 2

Determine the tangent, sine, and cosine ratios of any given acute angle.

In general terms, the three primary trigonometric ratios can be determined using one of two forms of notation. Study the example below.

Example 2

Give the three primary trigonometric ratios for each reference angle in right ΔOPQ.



Solution:

Line segment notation:

$$\sin\theta = \frac{PQ}{OP} \qquad \cos\alpha = \frac{QP}{OP}$$

$$\cos\theta = \frac{OQ}{OP} \qquad \sin\alpha = \frac{OQ}{OP}$$

$$\tan\theta = \frac{PQ}{OQ} \qquad \tan\alpha = \frac{OQ}{PQ}$$

Lower case notation:

$$\sin\theta = \frac{o}{q} \qquad \cos\alpha = \frac{o}{q}$$

$$\cos\theta = \frac{p}{q} \qquad \sin\alpha = \frac{p}{q}$$

$$\tan\theta = \frac{o}{p} \qquad \tan\alpha = \frac{p}{o}$$

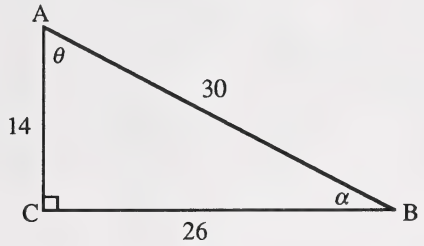
It is more meaningful to use numbers for the three primary trigonometric ratios. At times, these ratios are left as fractional numbers. The fractional number form is not too practical, so these fractions are changed to decimal numerals rounded to four decimal places.

The angles marked θ and α are the reference angles.

α is a Greek letter called alpha.

Example 3

Give the three primary trigonometric ratios for each acute reference angle in right triangle ABC. Use a fraction in simplest form as one ratio, and change each fraction to a decimal numeral rounded to four decimal places.



Solution:

$$\sin\theta = \frac{26}{30} = \frac{13}{15} = 0.8667$$

$$\cos\theta = \frac{14}{30} = \frac{7}{15} = 0.4667$$

$$\tan\theta = \frac{26}{14} = \frac{13}{7} = 1.8571$$

$$\sin\alpha = \frac{14}{30} = \frac{7}{15} = 0.4667$$

$$\cos\alpha = \frac{26}{30} = \frac{13}{15} = 0.8667$$

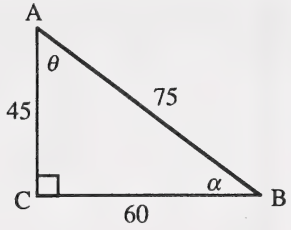
$$\tan\alpha = \frac{14}{26} = \frac{7}{13} = 0.5385$$

Now try the following questions.

Do either part a or part b of the following questions.

1. Give the three trigonometric ratios for each acute reference angle in the provided triangles. Provide two answers, one in simplest fraction form and the other in decimal number form rounded to four decimal places.

a.



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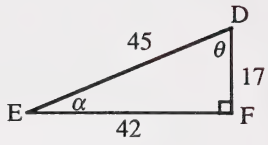
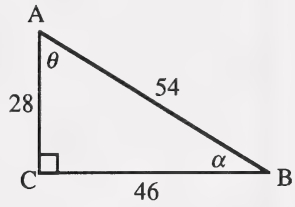
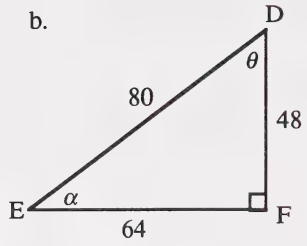


2. Give the three trigonometric ratios for each acute reference angle in the provided triangles. Provide two answers, one in simplified fraction form and the other in decimal number form rounded to four decimal places.

a.



b.



For solutions to **Activity 2**, turn to **Appendix A, Topic 3**.

The trigonometric values for any acute angle in a right triangle can be found by using a calculator. Study the example given.

Example 4

Use a table and a calculator to find the three trigonometric values for each of the following angles.

- 67°

Solution:

Calculator		Table
Enter	Display	
C	0	sin 67° ≐ 0.9205
67	67	
sin	0.920504853	
Enter	Display	
C	0	cos 67° ≐ 0.3907
67	67	
cos	0.390731128	
Enter	Display	
C	0	tan 67° ≐ 2.3559
67	67	
tan	2.355852366	

Partial Table of Trigonometric Values

Angle	sin θ	cos θ	tan θ
13°	0.2250	0.9744	0.2309
14°	0.2419	0.9703	0.2493
15°	0.2588	0.9659	0.2679
41°	0.6561	0.7547	0.8393
42°	0.6691	0.7431	0.9004
43°	0.6820	0.7314	0.9325
66°	0.9135	0.4076	2.2460
67°	0.9205	0.3907	2.3559
68°	0.9272	0.3746	2.4751

• 42°

Solution:

Calculator		Table
Enter	Display	
C	0	sin 42° ≐ 0.6691
42	42	
sin	0.669130606	
Enter	Display	
C	0	cos 42° ≐ 0.7431
42	42	
cos	0.743144825	
Enter	Display	
C	0	tan 42° ≐ 0.9004
42	42	
tan	0.900404044	

• 14°

Solution:

Calculator		Table
Enter	Display	
C	0	sin 14° ≐ 0.2419
14	14	
sin	0.241921895	
Enter	Display	
C	0	cos 14° ≐ 0.9703
14	14	
cos	0.970295726	
Enter	Display	
C	0	tan 14° ≐ 0.2493
14	14	
tan	0.249328002	

Now try the following questions using a calculator.

Do either question 3 or 4.

Give the three trigonometric ratios for each angle provided. Round each value to four decimal places.

3. a. 39° b. 63°

   c. 23°

4. a. 14° b. 53°

   c. 89°

For solutions to **Activity 2**, turn to **Appendix A, Topic 3**.

Activity 3

Determine the measure of an acute angle given the tangent, sine, or cosine ratio.

If one of the three primary trigonometric ratios is known, that particular reference angle can be found. This can be done by using a trigonometric values table or by using a calculator. Most mathematics books have trigonometric values tables toward the back of the book. For your use, there are tables provided in **Appendix B** at the end of this unit.

The example provided will show how any acute angle can be determined using the two methods mentioned above. If the ratio is a fraction, change the fraction to a decimal first.

Example 5

Find the measure of $\angle A$ to the nearest degree if the following values are given.

- $\sin A = 0.8913$
- $\cos A = 0.9644$
- $\tan A = 11.3467$

Solution:

Calculator	Table
• sin A = 0.8913 Enter: 0.8913 Press: Inv sin Result: 63.037 To the nearest degree, $\angle A \doteq 63°$.	0.8913 is between 0.8910 and 0.8988 or 63° and 64°. Differences: 0.8913 – 0.8910 = 0.0003 0.8988 – 0.8913 = 0.0075 0.8910 is closer to 0.8913. Therefore, $\angle A \doteq 63°$.
• cos A = 0.9644 Enter: 0.9644 Press: Inv cos Result: 15.334 To the nearest degree, $\angle A \doteq 15°$.	0.9644 is between 0.9613 and 0.9659 or 16° and 15°. Differences: 0.9644 – 0.9613 = 0.0031 0.9659 – 0.9644 = 0.0015 0.9644 is closer to 0.9659. Therefore, $\angle A \doteq 15°$.
• tan A = 11.3467 Enter: 11.3467 Press: Inv tan Result: 84.963 To the nearest degree, $\angle A \doteq 85°$.	11.3467 is between 9.5144 and 11.4301 or 84° and 85°. Differences: 11.3467 – 9.5144 = 1.8323 11.4301 – 11.3467 = 0.0834 11.3467 is closer to 11.4301. Therefore, $\angle A \doteq 85°$.

It is time to do some similar problems that follow.

Do either the odd- or even-numbered questions that follow.

$\doteq$ means approximately equal to.

These values were taken from the tables in **Appendix B**.

1. Use the calculator to find the angle for each of these ratios. Round each measure to the nearest degree.

   a. tan B = 2.3006
   b. cos B = 0.0659
   c. sin B = 0.5342
   d. tan B = 0.3542
   e. cos B = 0.7235
   f. sin B = 0.8993

2. Use a calculator to find the angle for each of these ratios. Round each measure to the nearest degree.

   a. sin C = 0.9644
   b. tan C = 1.4439
   c. tan C = 0.5336
   d. sin C = 0.3470
   e. cos C = 0.1142
   f. cos C = 0.7946

3. Use a trigonometric values table to find the angle measure for each of the following ratios. Round each measure to the nearest degree. Show all your work. The tables are found in **Appendix B** at the end of this unit.

   a. cos C = 0.2374
   b. tan C = 3.4009
   c. sin C = 0.7934

4. Use a trigonometric values table to find the angle measure of each of the following ratios. Round each measure to the nearest degree. Show all your work. The tables are found in **Appendix B** at the end of this unit.

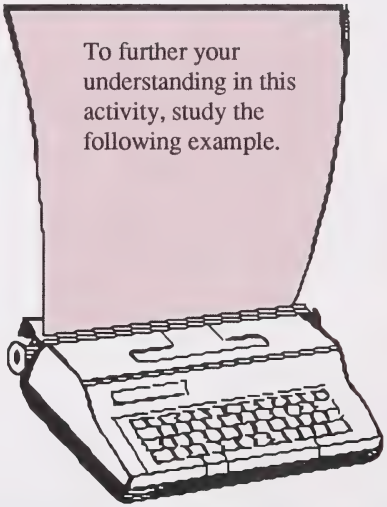
   a. sin D = 0.2084
   b. cos D = 0.9246
   c. tan D = 0.5539

For solutions to **Activity 3**, turn to **Appendix A, Topic 3**.

Activity 4

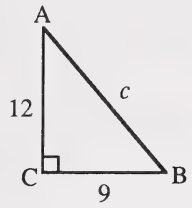
Determine the tangent, sine, or cosine ratios given two sides of a right triangle.

So far, you have examined how the three primary trigonometric ratios were found when the measures of the three sides of a right triangle were provided. All you had to do was compare the required sides for the reference angle. Suppose that only two sides were given. What would you do? The two sides would enable you to determine one or two trigonometric ratios, but not the others. When this happens, you must find the measure of the third side before attempting to find all three trigonometric ratios.



Example 6

Find the three trigonometric ratios for each acute angle in the triangles provided. Express the ratio as a simplified fraction and as a decimal numeral rounded to four decimal places. Round the side measure to the nearest whole number if necessary.

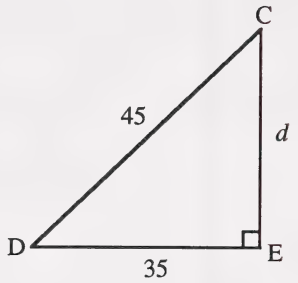


Solution:

Find c by using the Theorem of Pythagoras.

$$c^2 = a^2 + b^2$$
$$c^2 = 9^2 + 12^2$$
$$c^2 = 81 + 144$$
$$c^2 = 225$$
$$\sqrt{c^2} = \sqrt{225}$$
$$c = 15$$

$$\sin A = \frac{9}{15} \qquad \sin B = \frac{12}{15}$$
$$= \frac{3}{5} \qquad = \frac{4}{5}$$
$$= 0.6000 \qquad = 0.8000$$
$$\cos A = \frac{12}{15} \qquad \cos B = \frac{9}{15}$$
$$= \frac{4}{5} \qquad = \frac{3}{5}$$
$$= 0.8000 \qquad = 0.6000$$
$$\tan A = \frac{9}{12} \qquad \tan B = \frac{12}{9}$$
$$= \frac{3}{4} \qquad = \frac{4}{3}$$
$$= 0.7500 \qquad \doteq 1.3333$$



Solution:

Find d by using the Theorem of Pythagoras.

$$d^2 = e^2 - c^2$$
$$d^2 = 45^2 - 35^2$$
$$d^2 = 2025 - 1225$$
$$d^2 = 800$$
$$\sqrt{d^2} = \sqrt{800}$$
$$d \doteq 28.284$$
$$\doteq 28$$

$$\sin C = \frac{35}{45} = \frac{7}{9} \doteq 0.7778$$

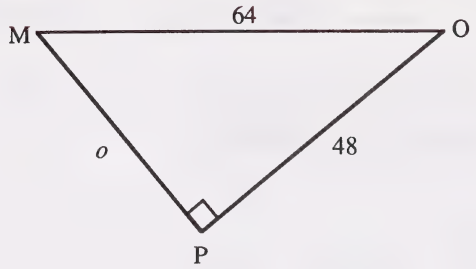
$$\sin D = \frac{28}{45} \doteq 0.6222$$

$$\cos C = \frac{28}{45} \doteq 0.6222$$

$$\cos D = \frac{35}{45} = \frac{7}{9} \doteq 0.7778$$

$$\tan C = \frac{35}{28} = \frac{5}{4} = 1.2500$$

$$\tan D = \frac{28}{35} = \frac{4}{5} = 0.8000$$



Solution:

Find o by using the Theorem of Pythagoras.

$$o^2 = p^2 - m^2$$
$$o^2 = 64^2 - 48^2$$
$$o^2 = 1792$$
$$\sqrt{o^2} = \sqrt{1792}$$
$$o \doteq 42.332$$
$$\doteq 42$$

The square of the hypotenuse is equal to the sum of the squares on the other two sides of a right triangle.

The measure for d is rounded to the nearest whole number.

The measure for o is rounded to the nearest whole number.

$\sin M = \frac{48}{64}$

$= \frac{3}{4}$

$= 0.7500$

$\sin O = \frac{42}{64}$

$= \frac{21}{32}$

$\doteq 0.6563$

$\cos M = \frac{42}{64}$

$= \frac{21}{32}$

$\doteq 0.6563$

$\cos O = \frac{48}{64}$

$= \frac{3}{4}$

$= 0.7500$

$\tan M = \frac{48}{42}$

$= \frac{8}{7}$

$\doteq 1.1429$

$\tan O = \frac{42}{48}$

$= \frac{7}{8}$

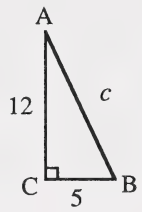
$= 0.8750$

You are now ready to do the rest of the exercises in this topic.

Do at least two of the parts in question 1.

Find the three primary trigonometric ratios for each acute angle in the given triangles. Express the side measures to the nearest whole number and express each trigonometric ratio as a fraction in simplest form and as a decimal rounded to four decimal places.

1. a.



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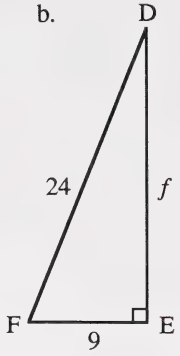
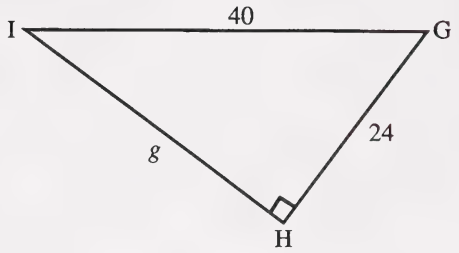
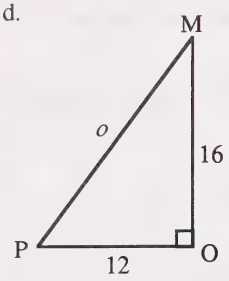


c.



d.



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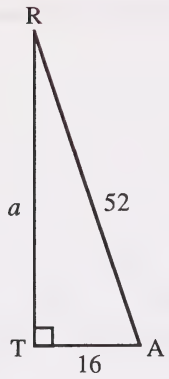

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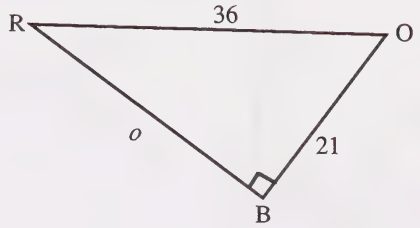

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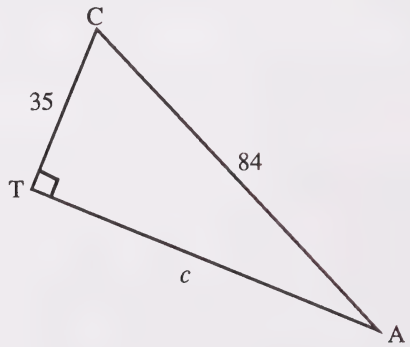

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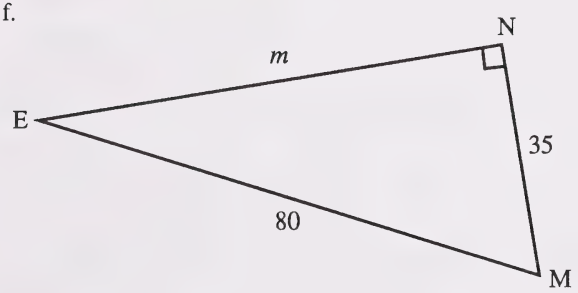

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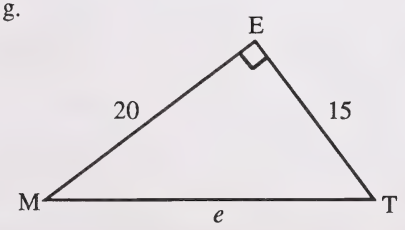


For solutions to **Activity 4**, turn to **Appendix A, Topic 3**.

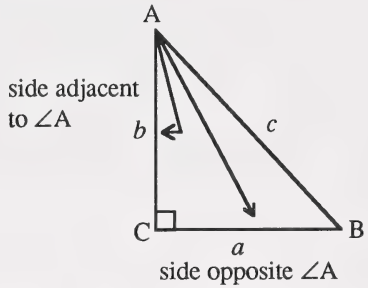
If you require help, do the Extra Help section.

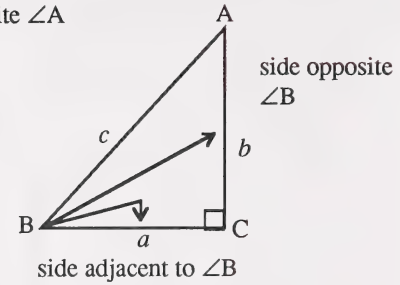
If you want more challenging explorations, do the Extensions section. } You may decide to do both.

Extra Help

When doing problems involving trigonometry, the following important points must be kept in mind.

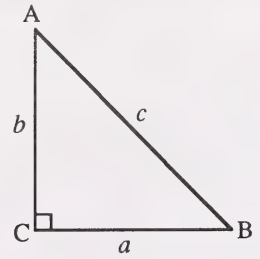
- The angle being considered or the angle which is used is called a reference angle.
- The two acute angles of a right triangle can be used as reference angles.
- The three primary trigonometric ratios are tangent (tan), sine (sin), and cosine (cos).
- The side opposite the right angle or the longest side of a right triangle is called the hypotenuse. The other two sides are the opposite side and the adjacent side depending on the reference angle used.





A right triangle has one angle which measures 90°. The symbol ⌞ is used to show where the right angle is. In ΔABC, ∠C is the right angle.

- In ΔABC, the primary trigonometric ratios for the acute angles $\angle A$ and $\angle B$ are shown below.

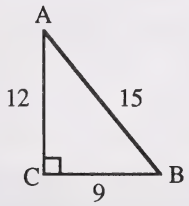


$\sin A = \frac{a}{c}$ $\quad$ $\sin B = \frac{b}{c}$

$\cos A = \frac{b}{c}$ $\quad$ $\cos B = \frac{a}{c}$

$\tan A = \frac{a}{b}$ $\quad$ $\tan B = \frac{b}{a}$

- The three trigonometric ratios can be expressed as fractions in simplest form or as decimal numerals rounded to four decimal places. See the following example.



$\sin A = \frac{9}{15}$ $\quad$ $\sin B = \frac{12}{15}$

$= \frac{3}{5}$ $\quad$ $= \frac{4}{5}$

$= 0.6000$ $\quad$ $= 0.8000$

$\cos A = \frac{12}{15}$ $\quad$ $\cos B = \frac{9}{15}$

$= \frac{4}{5}$ $\quad$ $= \frac{3}{5}$

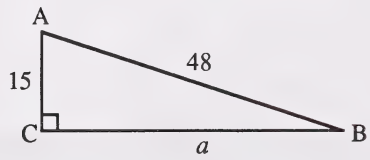
$= 0.8000$ $\quad$ $= 0.6000$

$\tan A = \frac{9}{12}$ $\quad$ $\tan B = \frac{12}{9}$

$= \frac{3}{4}$ $\quad$ $= \frac{4}{3}$

$= 0.7500$ $\quad$ $\doteq 1.3333$

- When two sides of a right triangle are given, the third side can be calculated using the Theorem of Pythagoras. Once this is done, the three primary trigonometric ratios can be found using fractions or decimal numerals. Study the following example. Other situations can occur.



To help you remember the trigonometric relations, use the following memory aid.

$$\left.\begin{matrix}S\\O\\H\end{matrix}\right\} \sin = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$= \frac{O}{H}$$

$$\left.\begin{matrix}C\\A\\H\end{matrix}\right\} \cos = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$= \frac{A}{H}$$

$$\left.\begin{matrix}T\\O\\A\end{matrix}\right\} \tan = \frac{\text{side opposite}}{\text{side adjacent}}$$

$$= \frac{O}{A}$$

Perhaps this phrase will help!

SOAK A TOE.

$$c^2 = a^2 + b^2$$
$$48^2 = a^2 + 15^2$$
$$a^2 = 48^2 - 15^2$$
$$a^2 = 2304 - 225$$
$$a^2 = 2079$$
$$\sqrt{a^2} = \sqrt{2079}$$
$$a \doteq 45.596$$
$$a \doteq 46$$

$$\sin A = \frac{46}{48} = \frac{23}{24} \doteq 0.9583 \qquad \sin B = \frac{15}{48} = \frac{5}{16} = 0.3125$$

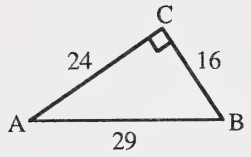
$$\cos A = \frac{15}{48} = \frac{5}{16} = 0.3125 \qquad \cos B = \frac{46}{48} = \frac{23}{24} \doteq 0.9583$$

$$\tan A = \frac{46}{15} \doteq 3.0667 \qquad \tan B = \frac{15}{46} \doteq 0.3261$$

Now you can put all of these points to use by doing the exercises that follow.

1. Give the three trigonometric ratios for each acute angle. If possible, simplify each fraction, but leave your answer as a fraction.

a.



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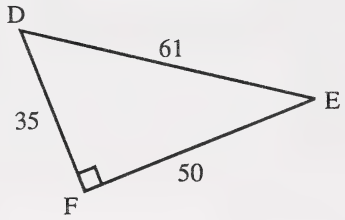
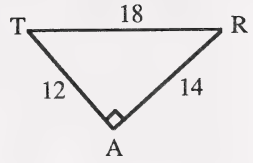
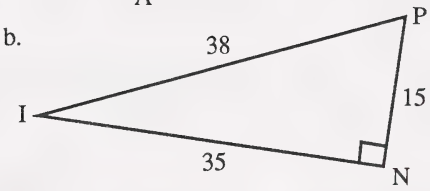


2. Give three trigonometric ratios for each acute angle. Express each value rounded to four decimal places.

a.

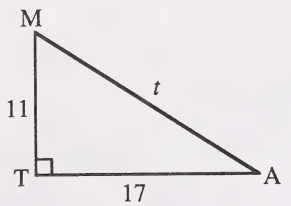


b.



3. In each of the following right triangles, find the length of the third side, and then calculate the three trigonometric ratios for each acute angle rounded to four decimal places. If necessary, round the side measure to the nearest whole number.

a.



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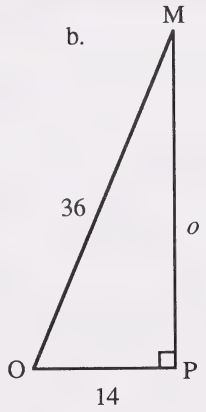

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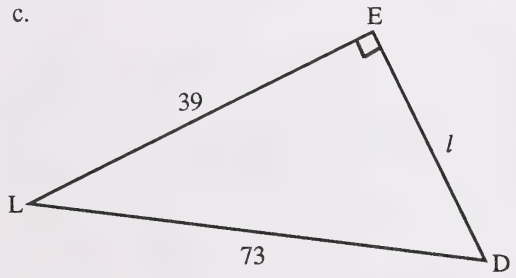


For solutions to **Extra Help**, turn to **Appendix A, Topic 3**.

Extensions

By now you know that the three trigonometric ratios for any acute angle of a right triangle are as follows.

$$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}} \qquad s = \frac{O}{H} \rightarrow \text{SOH}$$

$$\cos\theta = \frac{\text{side adjacent}}{\text{hypotenuse}} \qquad c = \frac{A}{H} \rightarrow \text{CAH}$$

$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} \qquad t = \frac{O}{A} \rightarrow \text{TOA}$$

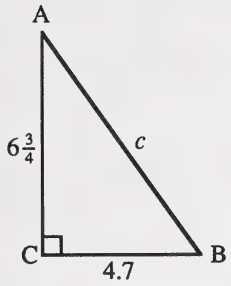
To this point, you used whole numbers for the side measures of a right triangle.

In this section, you will be asked to use fractions, decimal numerals, and radicals for the measures of the sides. No matter what type of number system is used, the primary trigonometric ratios can be found and are usually rounded to four decimal places. Approximation for the unknown side measures is often needed. The type of estimation is dictated by the given value.

It is also possible to have a mixture of known values. If this happens, change all the values to whole numbers or decimal numerals before applying them to the trigonometric ratios.

Example 7

Find the trigonometric ratios for each acute angle in each of the following triangles. Round your answer to four decimal places.



Solution:

$$c^2 = a^2 + b^2$$
$$c^2 = \left(6\tfrac{3}{4}\right)^2 + (4.7)^2$$
$$c^2 = (6.75)^2 + (4.7)^2$$
$$c^2 = 45.5625 + 22.09$$
$$c^2 = 67.6525$$
$$\sqrt{c^2} = \sqrt{67.6525}$$
$$c \doteq 8.2251$$
$$c \doteq 8.2$$

Ratios for $\angle A$

$$\sin A = \frac{4.7}{8.2} \doteq 0.5732$$

$$\cos A = \frac{6.75}{8.2} \doteq 0.8232$$

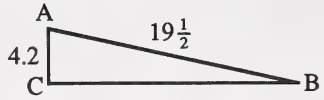
$$\tan A = \frac{4.7}{6.75} \doteq 0.6963$$

Ratios for $\angle B$

$$\sin B = \frac{6.75}{8.2} \doteq 0.8232$$

$$\cos B = \frac{4.7}{8.2} \doteq 0.5732$$

$$\tan B = \frac{6.75}{4.7} \doteq 1.4362$$



Solution:

$$c^2 = a^2 + b^2$$
$$(19.5)^2 = a^2 + (4.2)^2$$
$$380.25 = a^2 + 17.64$$
$$a^2 = 380.25 - 17.64$$
$$a^2 = 362.61$$
$$\sqrt{a^2} = \sqrt{362.61}$$
$$a \doteq 19.0423$$
$$a \doteq 19.0$$

Ratios for $\angle A$

$$\sin A = \frac{19.0}{19.5} \doteq 0.9744$$

$$\cos A = \frac{4.2}{19.5} \doteq 0.2154$$

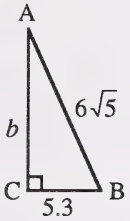
$$\tan A = \frac{19.0}{4.2} \doteq 4.5238$$

Ratios for $\angle B$

$$\sin B = \frac{4.2}{19.5} \doteq 0.2154$$

$$\cos B = \frac{19.0}{19.5} \doteq 0.9744$$

$$\tan B = \frac{4.2}{19.0} \doteq 0.2211$$



Solution:

$$c^2 = a^2 + b^2$$

$$\left(6\sqrt{5}\right)^2 = (5.3)^2 + b^2$$

$$180 = 28.09 + b^2$$

$$b^2 = 180 - 28.09$$

$$b^2 = 151.91$$

$$\sqrt{b^2} = \sqrt{151.91}$$

$$b \doteq 12.3$$

Ratios for $\angle A$

$$\sin A = \frac{5.3}{6\sqrt{5}} = \frac{5.3}{13.4} \doteq 0.3955$$

$$\cos A = \frac{12.3}{6\sqrt{5}} = \frac{12.3}{13.4} \doteq 0.9179$$

$$\tan A = \frac{5.3}{12.3} \doteq 0.4309$$

Ratios for $\angle B$

$$\sin B = \frac{12.3}{6\sqrt{5}} = \frac{12.3}{13.4} \doteq 0.9179$$

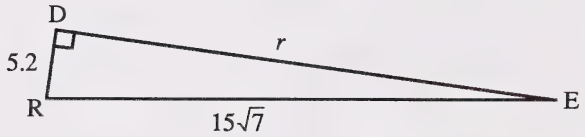
$$\cos B = \frac{5.3}{6\sqrt{5}} = \frac{5.3}{13.4} \doteq 0.3955$$

$$\tan B = \frac{12.3}{5.3} \doteq 2.3208$$

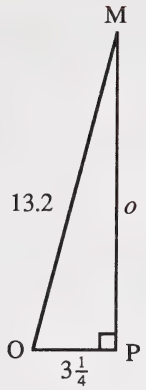
Do the following exercises.

1. Calculate the primary ratios for each acute angle in the following right triangles. Round your answers to four decimal places.

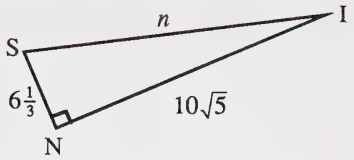
a.



b.

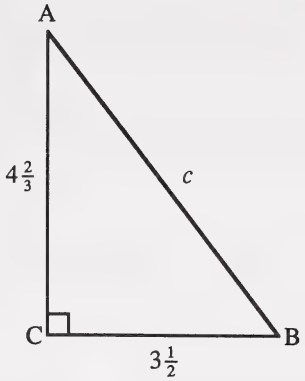


d.



For solutions to **Extensions**, turn to **Appendix A, Topic 3**.

c.



Topic 4 Solving Right Triangles

Introduction

One fine summer day, Colin and his friend Jason took the gondola to the top of Sulphur Mountain in Banff National Park. From the top, they were able to see the Banff townsite. A guide told them that the height of Sulphur Mountain is about 2450 m. They were also told that the angle of depression to the townsite was 30°. Using these facts, the boys were able to calculate the distance from the bottom of the mountain to the townsite and the distance from the top of the mountain to the townsite. Do you think you can do this? The material in this topic should help you do this. The actual solution to finding these distances will be shown in the **Extensions** section of this topic.

What Lies Ahead

Throughout the topic you will learn to

1. determine the measure of an acute angle when two sides are known
2. determine the length of a side when the measure of one side and one acute angle are known
3. solve right triangles
4. solve word problems involving right triangles

Now that you know what to expect, turn the page to begin your study of solving right triangles.

Exploring Topic 4

Activity 1

Determine the measure of an acute angle when two sides are known.

When working with right-angle triangles, you have seen that many different situations can exist. The following are a few examples of such situations.

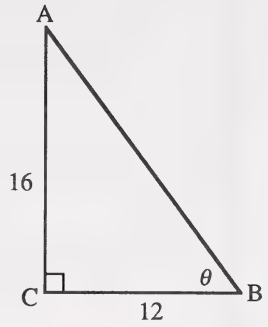
- Find the third side when the measured two sides are given. This is done by using the Theorem of Pythagoras.
- Find the measure of an acute angle when the measure of two sides are given. Use the appropriate trigonometric ratio to find the angle measurement.
- Find the measure of a side when the measure of one side and one acute angle is known. Use the appropriate trigonometric ratio to find the measurement.

You have applied the Theorem of Pythagoras earlier, so very little time will be spent on this concept. You will use it later as required.

The first situation you will deal with is finding the measure of an acute angle when the measures of two sides are given. To do this, you can use a trigonometric table or a calculator. Both methods will be shown. The following examples illustrate the various arrangements which are commonly used.

Example 1

Find the measure of the indicated acute angle in each right triangle. Express each measure to the nearest degree.



Solution:

Choose the appropriate trigonometric ratio. In this case, use the tangent ratio.

$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}}$$

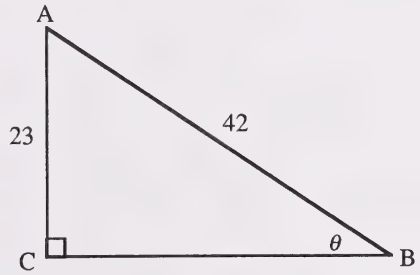
$$\tan\theta = \frac{16}{12}$$

$$= \frac{4}{3}$$

$$\doteq 1.3333$$

Calculator		Table
Enter	Display	1.3333 is between 1.3270 and 1.3764 or 53° and 54°.
C	0	Differences: 1.3333 – 1.3270 = 0.0063
1.3333	1.3333	1.3764 – 1.3333 = 0.0431
Inv tan	53.1294148	1.3270 is closer to 1.3333.
To the nearest degree, $\theta = 53°$.		Therefore, $\theta = 53°$ to the nearest degree.

Use the table in **Appendix B** at the back of this unit.



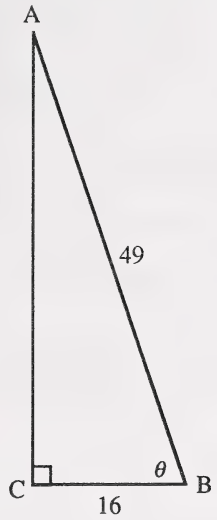
Solution:

Choose the appropriate trigonometric ratio. In this case, use the sine ratio.

$$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{23}{42}$$
$$\doteq 0.5476$$

Calculator		Table
Enter	Display	0.5476 is between 0.5446 and 0.5592 or 33° and 34°.
C	0	Differences: 0.5476 – 0.5446 = 0.0030
0.5476	0.5476	0.5592 – 0.5476 = 0.0116
Inv sin	33.20251823	0.5446 is closer to 0.5476.
To the nearest degree, $\theta = 33°$.		Therefore, $\theta = 33°$ to the nearest degree.



Solution:

Choose the appropriate trigonometric ratio. In this case, use the cosine ratio.

$$\cos\theta = \frac{16}{49}$$

$$\doteq 0.3265$$

$$\cos\theta = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

Calculator		Table
Enter	Display	0.3265 is between 0.3256 and 0.3420 or 71° and 70°.
C	0	Differences: 0.3265 – 0.3256 = 0.0009
0.3265	0.3265	0.3420 – 0.3265 = 0.0155
Inv cos	70.94352318	0.3256 is closer to 0.3265.
To the nearest degree, $\theta = 71°$.		Therefore, $\theta = 71°$ to the nearest degree.

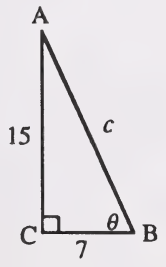
If you were asked to find the measure of the other acute angle, your procedure would be similar to what was done here. The ratio for the sine and cosine would be interchanged and the ratio for the tangent would be the reciprocal of the original one.

Try the following problems in the exercise which follows.

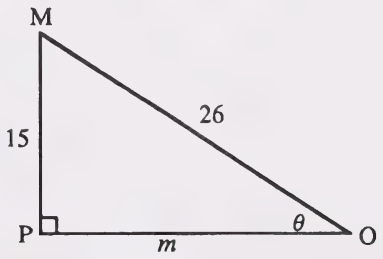
Do any two parts from 1, 2, and 3, and do all of question 4.

1. Find the measure of θ to the nearest degree. Use a calculator.

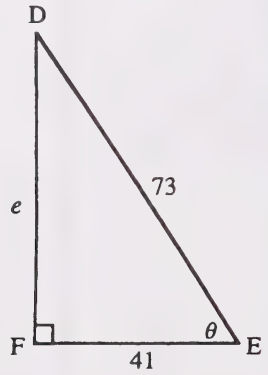
a.



b.

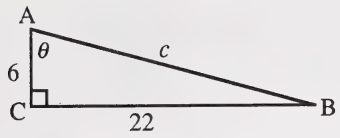


c.

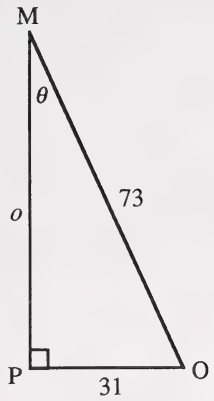


2. Find the measure of θ to the nearest degree. Use a calculator.

a.



b.



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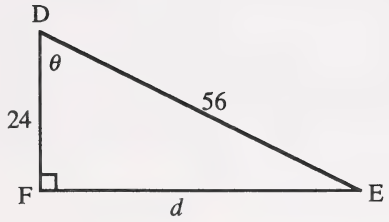
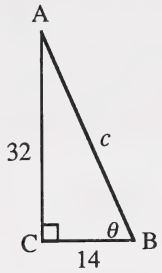
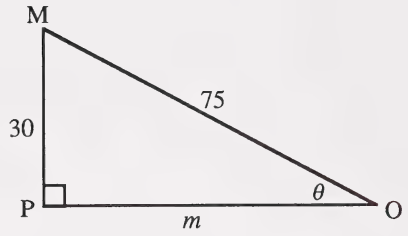


3. Find the measure of θ to the nearest degree. Use a calculator.

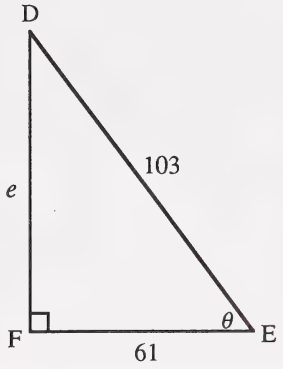
a.



b.

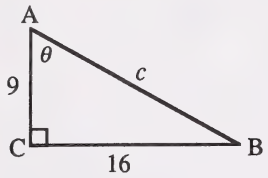


c.

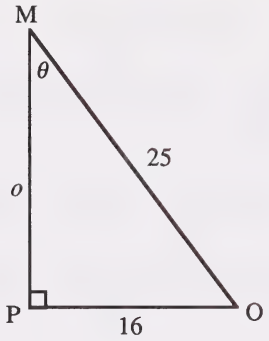


4. Find the measure of θ to the nearest degree. Use a trigonometric table.

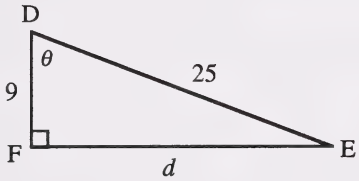
a.



b.



c.



For solutions to **Activity 1**, turn to **Appendix A, Topic 4**.

Activity 2

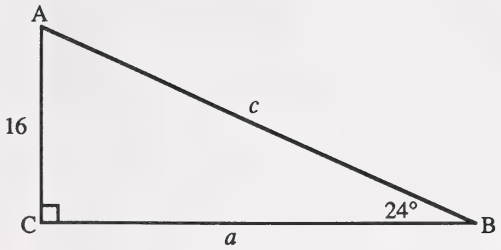
Determine the length of a side when the measure of one side and one acute angle are known.

This can be done by using a calculator or a trigonometric table. The example that follows shows the various adaptations which are commonly found.

Example 2

Find the measure of the side indicated in each of the following right triangles. Express your answer to the nearest tenth.

- Find a.



Solution:

To find a, use the tangent ratio.

$$\tan 24° = \frac{16}{a}$$

$$\tan 24° \times a = 16$$

$$a = \frac{16}{\tan 24°}$$

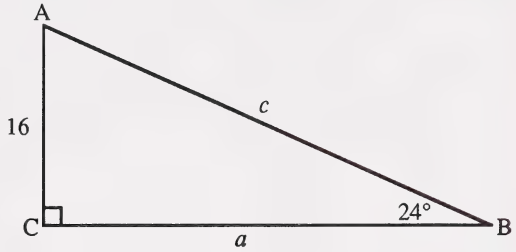
$$a = \frac{16}{0.4452}$$

$$a \doteq 35.939$$

$$a \doteq 35.9 \text{ units}$$

The measure of side a is approximately 35.9 units.

- Find c.



Solution:

To find c, use the sine ratio.

$$\sin 24° = \frac{16}{c}$$

$$\sin 24° \times c = 16$$

$$c = \frac{16}{\sin 24°}$$

The calculator steps to solve for a in $a = \dfrac{16}{\tan 24°}$ are as follows:

Enter	Display
C	0
16	16
÷	16
24	24
tan	0.445228685
=	35.93658838

The measure of a is 35.9 units, to the nearest tenth.

To use the trigonometric table, find tan 24° in the tan column and divide 16 by this value. Finally, round off to one decimal place.

$$a = 16 \div 0.4452$$

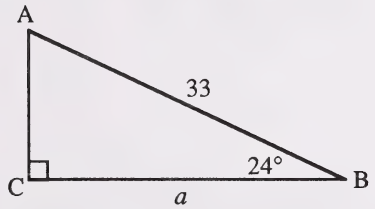
$$a \doteq 35.939$$

$$a \doteq 35.9 \text{ units}$$

Calculator		Table
Enter	Display	
C	0	
16	16	
÷	16	
24	24	$c = 16 \div \sin 24°$
sin	0.406736643	$c = 16 \div 0.4067$
=	39.33749337	$c \doteq 39.341$
		$c \doteq 39.3$ units
To one decimal place, the measure of side c is 39.3 units.		The measure of c is approximately 39.3 units.

Calculator		Table
Enter	Display	
C	0	
33	33	
×	33	
24	24	$a = 33 \times \cos 24°$
cos	0.913545457	$a = 33 \times 0.9135$
=	30.14700011	$a \doteq 30.1455$
		$a \doteq 30.1$
To one decimal place, the measure of side a is 30.1 units.		The measure of a is approximately 30.1 units.

- Find a.



Solution:

To find a, use the cosine ratio.

$$\cos 24° = \frac{a}{33}$$

$$a = 33 \times \cos 24°$$

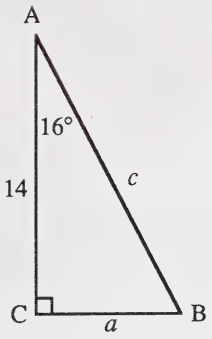
From these examples, you should conclude that to solve for the designated side, it is important to choose the correct trigonometric ratio when making the equation. Once this is done, you either multiply or divide to get the correct answer.

Try some of the following exercises.

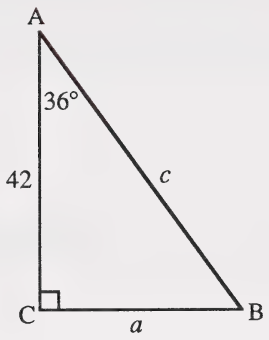
Do the odd- or the even-numbered questions from the following. Use a calculator and round your answer to the nearest hundredth or to two decimal places.

1. Solve for a.

a.

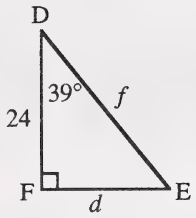


b.

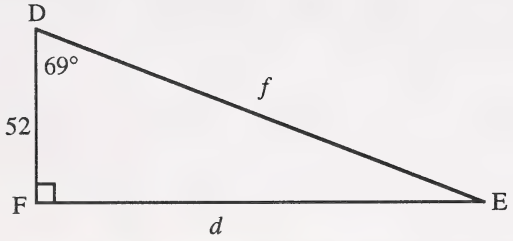


2. Solve for d.

a.

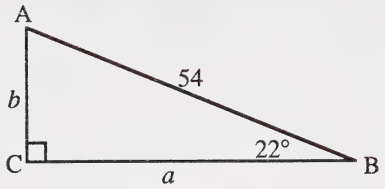


b.

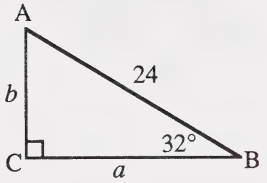


3. Solve for b.

a.

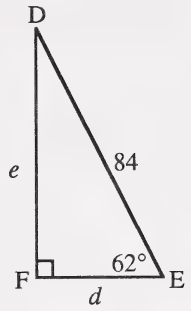


b.

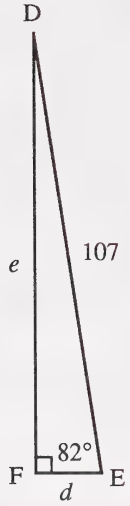


4. Solve for e.

a.

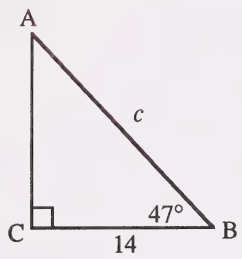


b.

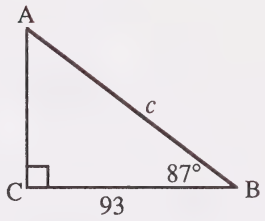


5. Solve for c.

a.

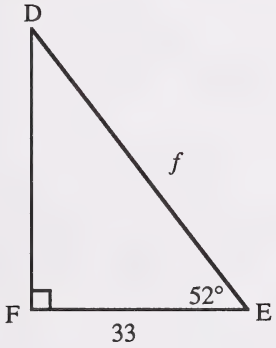


b.

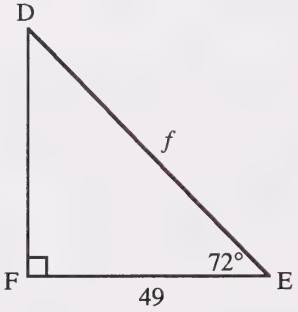


6. Solve for f.

a.



b.



For solutions to **Activity 2**, turn to **Appendix A**, **Topic 4**.

You may study the next two activities by doing either **Part A** or **Part B** or both. **Part A** covers the objective with an audiotape while **Part B** covers the topic through the print mode. Whichever way you choose to study this topic, do the questions at the end of **Part B**.

Activity 3

Solve right triangles.

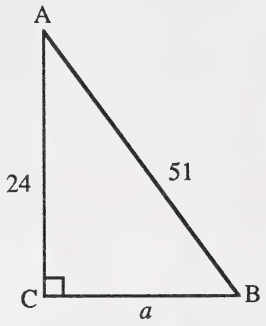
Part A

Audio Activity

Insert the audiotape titled *Mathematics 23 - Unit 7 - Trigonometry* into your tape recorder and follow the instructions on the tape.

1 Solve right triangle ABC.

Find ∠A, ∠B, and side *a*.



2 Solving for ∠A

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos A = \dfrac{24}{51}$

Using your calculator, enter the following to complete the blanks.

Enter 24, Press [÷], Enter 51, Press [=]

$\cos A =$ _____ Press [Inv], Press [cos]

$\angle A =$ _____ Result

$\angle A \doteq$ _____ to the nearest degree

3 Solving for ∠B

$\sin B = \dfrac{\text{opposite}}{\text{hypotenuse}}$ Use your calculator.

Enter 24, Press [÷], Enter 51, Press [=]

$\sin B = \dfrac{24}{51}$

$\sin B =$ _____ Press [Inv], Press [sin]

$\angle B =$ _____

$\angle B \doteq$ _____

4

$$\angle A + \angle B + \angle C = 180°$$
$$62° + \angle B + 90° = 180°$$
$$\angle B + 152° = 180°$$
$$\angle B = 180° - 152°$$
$$\angle B = 28°$$

5

$\sin A = \dfrac{a}{51} \rightarrow a = 51 \sin 62° \rightarrow a = 51(\quad) \rightarrow a =$ _____

$\tan A = \dfrac{a}{24} \rightarrow a = 24 \tan 62° \rightarrow a = 24(\quad) \rightarrow a =$ _____

$\cos B = \dfrac{a}{51} \rightarrow a = 51 \cos 28° \rightarrow a = 51(\quad) \rightarrow a =$ _____

$\tan B = \dfrac{24}{a} \rightarrow a = \dfrac{24}{\tan 28°} \rightarrow a = 24 \div (\quad) \rightarrow a =$ _____

6

$$c^2 = a^2 + b^2$$
$$51^2 = a^2 + 24^2$$
$$a^2 = 51^2 - 24^2$$
$$a^2 = 2601 - 576$$
$$a^2 = 2025$$
$$a = 45$$

7 Summary:

$\angle A = 62°$	side $a = 45$
$\angle B = 28°$	side $b = 24$
$\angle C = 90°$	side $c = 51$

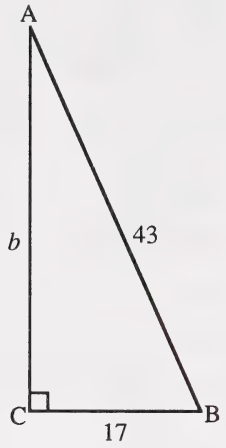
Part B

From what you have studied previously, you know that the measure of any acute angle in a right triangle can be found if the measures of any two sides are provided. Similarly, the measure of any side can be found if the measure of one acute angle and the measure of one side is given.

Generally speaking, when you are asked to find the missing measures of unknown angles and the missing measures of unknown sides, you are solving a right triangle. When you are asked to solve a right triangle, be sure to choose and use the appropriate trigonometric ratio. Often, one of the angles is found using another trigonometric ratio or the fact that the sum of the three angles of any triangle is 180°. Similarly, when one of the two missing side measures is found, the third side can be found by using another trigonometric ratio or the Theorem of Pythagoras. Study the following example to see how all of this applies. Notice that an alternative method is also shown.

Example 3

Solve right triangle ABC. Express all values to the nearest whole number.

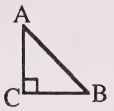


Solution:

Solve for ∠B.
Use the cosine ratio.

$$\cos B = \frac{17}{43}$$
$$\doteq 0.3953$$

In right ΔABC,



$\angle A + \angle B + \angle C = 180°$.

Enter	Display
C	0
.3953	0.3953
Inv cos	66.7153144

To the nearest degree, $\angle B = 67°$.

Solve for $\angle A$.
Use the sine ratio.

$$\sin A = \frac{17}{43}$$
$$= 0.3953$$

Enter	Display
C	0
.3953	0.3953
Inv sin	23.2846856

To the nearest degree, $\angle A = 23°$.

The measure for $\angle A$ can also be found as follows.

$$\angle A + \angle B + \angle C = 180°$$
$$\angle A + 67° + 90° = 180°$$
$$\angle A + 157° = 180°$$
$$\angle A = 180° - 157°$$
$$\angle A = 23°$$

To find side b, you can use any one of the following trigonometric ratios.

$$\tan B = \frac{b}{17} \rightarrow \tan 67° = \frac{b}{17} \rightarrow b = 17 \times 2.3559 \rightarrow b = 40$$

$$\sin B = \frac{b}{43} \rightarrow \sin 67° = \frac{b}{43} \rightarrow b = 43 \times 0.9205 \rightarrow b = 40$$

$$\tan A = \frac{17}{b} \rightarrow \tan 23° = \frac{17}{b} \rightarrow b = 17 \div 0.4245 \rightarrow b = 40$$

$$\cos A = \frac{b}{43} \rightarrow \cos 23° = \frac{b}{43} \rightarrow b = 43 \times 0.9205 \rightarrow b = 40$$

In each case, the final answer is $b = 40$.

You can also use the Theorem of Pythagoras to solve for b.

$$c^2 = a^2 + b^2$$
$$43^2 = 17^2 + b^2$$
$$1849 = 289 + b^2$$
$$b^2 = 1849 - 289$$
$$b^2 = 1560$$
$$\sqrt{b^2} = \sqrt{1560}$$
$$b \doteq 39.5$$
$$b \doteq 40$$

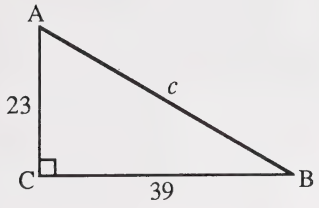
The measure of side b is approximately 40 units.

Summary:

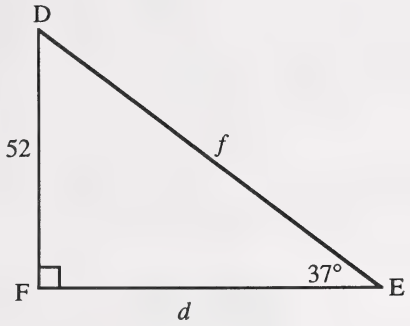
$\angle A = 23°$	side a = 17 units
$\angle B = 67°$	side b = 40 units
$\angle C = 90°$	side c = 43 units

Try three of the following five questions.

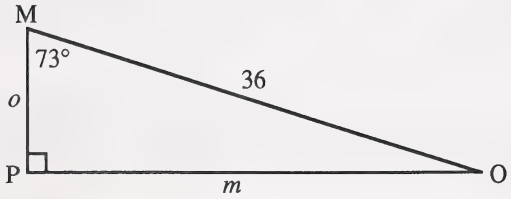
1. Solve ΔABC.



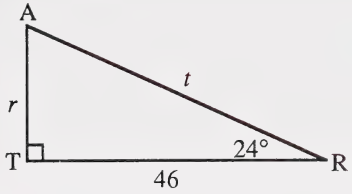
2. Solve for ΔDEF.



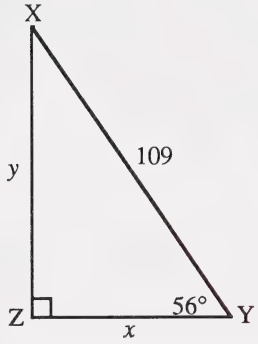
3. Solve ΔMOP.



4. Solve ΔART.



5. Solve ΔXYZ.



For solutions to **Activity 3**, turn to **Appendix A, Topic 4**.

Activity 4

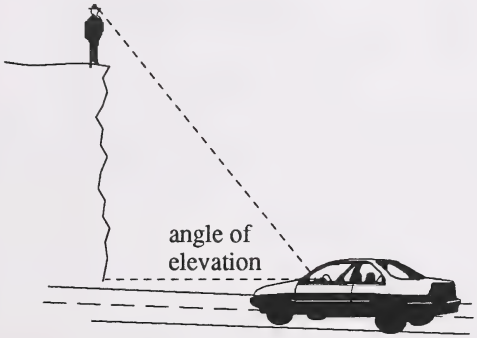
Solve word problems involving right triangles.

Part A

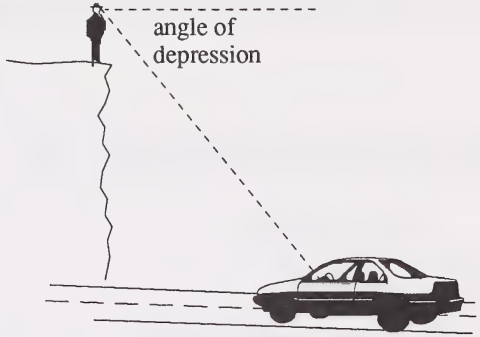
Audio Activity

Continue with the tape titled *Mathematics 23 - Unit 7 - Trigonometry*. Complete the exercises at the end of **Part B** after you have listened to the tape.

8



9



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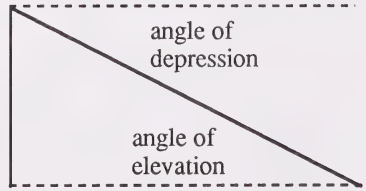

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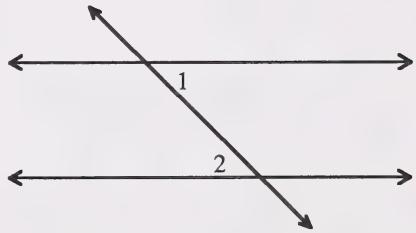
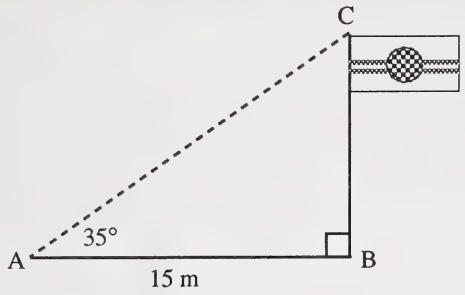


12



13

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 35° = \frac{\text{opposite}}{15}$$

$$\text{opposite} = 15 \tan 35°$$

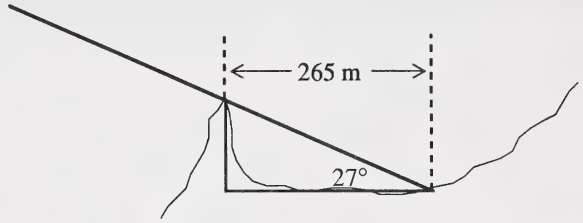
$$= 15(0.700\,207\,538)$$

$$= 10.503\,113\,07$$

$$= 10.5 \text{ m}$$

The height of the flagpole is 10.5 m.

14



15

$$\tan 27° = \frac{\text{opposite}}{265}$$

$$\text{opposite} = 265 \tan 27°$$

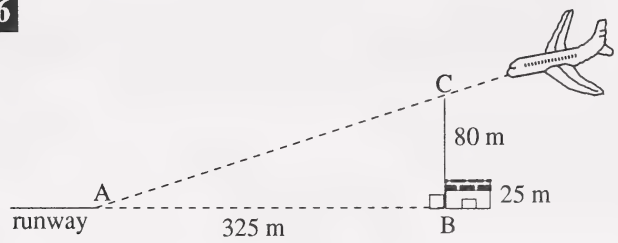
$$= 265(0.509\,525\,449)$$

$$= 135.024\,244\,1$$

$$= 135 \text{ m}$$

The height of the rim is 135 m.

16



17

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

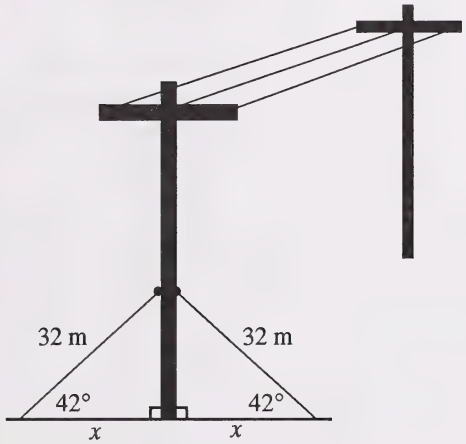
$$\tan A = \frac{105}{325}$$

$$\tan A = 0.323\,076\,923$$

$$\angle A = 17.904\,447\,51$$

$$\angle A = 18°$$

18



19

$$\cos 42° = \frac{x}{32}$$

$$x = 32 \cos 42°$$

$$= 32(0.743\,144\,825)$$

$$= 23.780\,634\,42$$

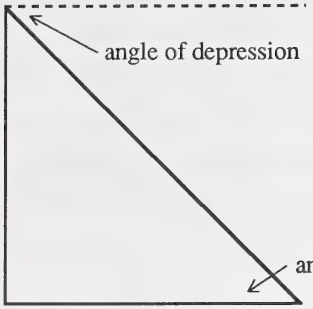
$$x = 24 \text{ m}$$

$$2x = 48 \text{ m}$$

Part B

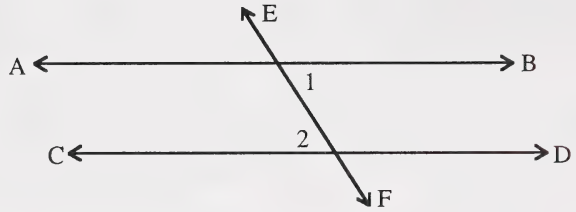
It is now time to apply all of your trigonometric skills to solve problems which may be in diagram form or in written form. When dealing with a word problem, always start your solution with a simple sketch. Label all the known parts and determine what needs to be found.

Before doing an example or two, you must know the difference between an angle of depression and an angle of elevation. This difference is illustrated by the following diagram.



These are related angles of depression and elevation.

It is important to keep in mind that related angles of depression and elevation are equal in measure. This is true because a transversal intersects two parallel lines.



Angles 1 and 2, which are alternate angles, can be compared to related angles of depression and elevation.

Since $\angle 1 = \angle 2$, the angle of depression is equal to the angle of elevation.

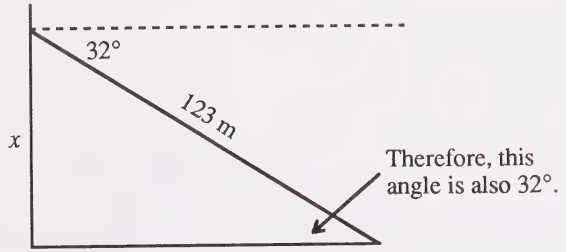
angle of depression

angle of elevation

Study the following examples.

Example 4

Solve for x in the following sketch.
Round to two decimal places.



Solution:

Remember that the angle of depression is equal to the angle of elevation.

$$\sin 32° = \frac{x}{123}$$

$$x = 123 \times \sin 32°$$

$$x = 123 \times 0.5299$$

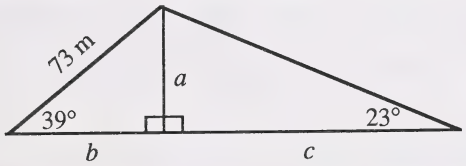
$$x \doteq 65.1777$$

$$x \doteq 65.18$$

The value for x is 65.18 m.

Example 5

Shelley and her brother Kellie decided to fly a kite on a particular windy day. Shelley held the string which stretched out to 73 m long. The angle that this string made with the ground was 39°. Kellie estimated that the angle of elevation from his viewpoint was 23°. Calculate the distance between where Shelley and Kellie were standing. Round the answer to the nearest metre.



Make a simple sketch and enter all the provided measurements.

Solution:

Step 1: Find the measure for a.

$$\sin 39° = \frac{a}{73}$$

$$a = 73 \times 0.6293$$

$$a = 45.94$$

$$a \doteq 46$$

The measure for a is 46 m.

Step 2: Find the measure for b.

$$\tan 39° = \frac{a}{b}$$

$$0.8098 = \frac{46}{b}$$

$$b = 46 \div 0.8098$$

$$b \doteq 56.80$$

$$b \doteq 57$$

The measure for b is 57 m.

Step 3: Find the measure for c.

$$\tan 23° = \frac{46}{c}$$

$$0.4245 = \frac{46}{c}$$

$$c = 46 \div 0.4245$$

$$c \doteq 108.36$$

$$c \doteq 108 \text{ m}$$

The measure for c is 108 m.

Step 4: Add the b and c values.

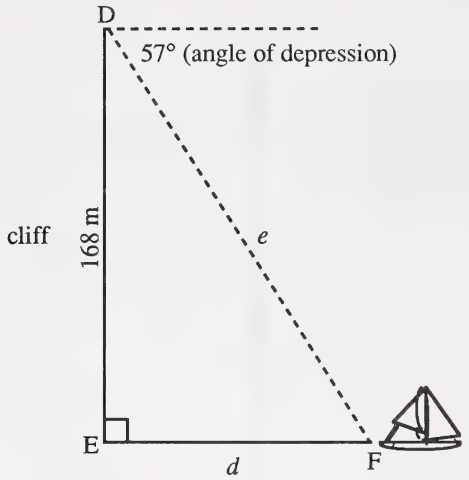
$$57 \text{ m} + 108 \text{ m} = 165 \text{ m}$$

The total distance between the points where Shelley and Kellie are standing is 165 m.

Do the problems in the exercise that follows. Remember to start each solution with a diagram if one is not provided.

Do any six of the following questions.

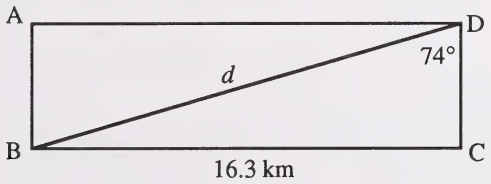
1. How far is the sailboat from the base of the cliff? The angle of depression is 57°. Round the answer to the nearest metre.



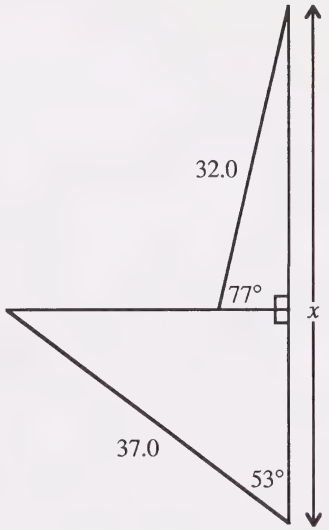
2. A flagpole is held in a vertical position by two guy wires, each 42 m in length. If the wires make an angle of 69° with the ground, find the horizontal distance along the ground between the two wires. Round the answer to one decimal.

3. An observer 1.7 m tall finds the angle of elevation to the top of a cliff is 57°. If the observer is 54 m from the cliff's base, find the height of the cliff. Round the answer to one decimal place.

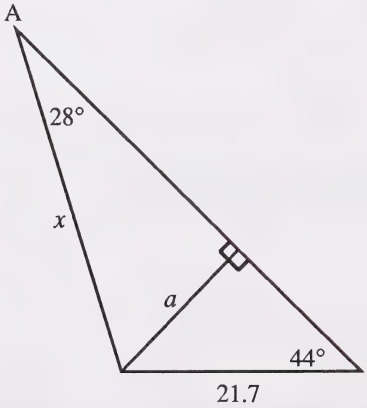
4. Find the length of the diagonal in this diagram. Round the answer to one decimal place.



5. Find x rounded to one decimal place.



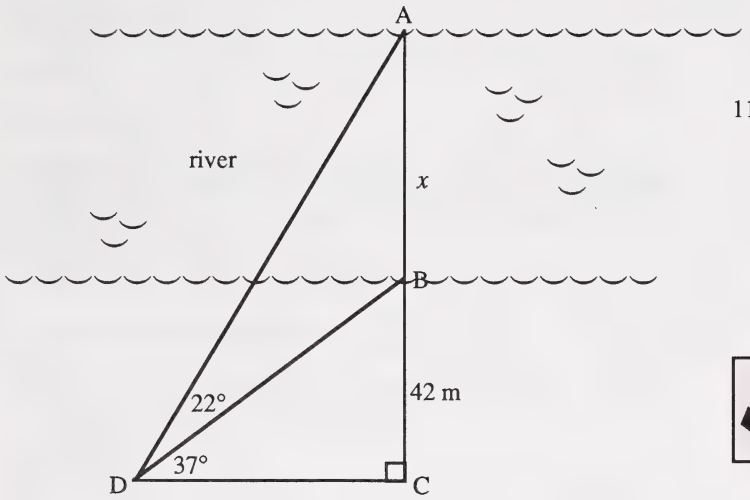
6. Find x rounded to one decimal place.



A diagonal joins two nonconsecutive vertices of a polygon.

7. Blooper and Slinky are climbing up a hill which is inclined at 23° to the horizontal. After they have walked 1016 m up the slope, find their height above the horizontal. Round the answer to the nearest metre.

8. Find the width of the river to the nearest metre.



9. A ladder 4.7 m long is leaned against a wall and makes an angle of 47° with the ground.

   Find the following:
   a. the distance from the foot of the ladder to the wall
   b. the distance the ladder reaches up the wall

   Round the answers to one decimal place.

10. A radio tower 163 m tall is anchored by a guy wire. The wire makes an angle of 64° with the ground. A maintenance order suggests that the base of the guy wire should be moved an additional 14 m from the base. Find the following rounded to the nearest whole number.

    a. How much longer is the new wire?
    b. Find the angle which will be made between the new wire and the ground.

11. Two trees are 137 m apart. From a point halfway between the two trees, the angle of elevation to their tops is 23° and 33°. How much taller is one tree than the other? Round the answer to the nearest metre.

For solutions to **Activity 4**, turn to **Appendix A, Topic 4**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section. } You may decide to do both.

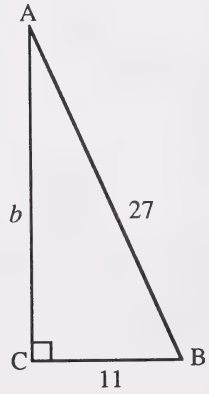
Extra Help

When you are asked to solve a right triangle, you must do the following.

- Find the measure of the unknown angle or angles.
- Find the measure of the unknown side or sides.
- Approximate the angle measures to the nearest degree.
- Approximate the side measures according to the specific directions which are usually given. If no directions are provided, round the answer so that they are of the same accuracy as the given values.
- During the solving procedures, choose a method which is most meaningful. This means that there may be several ways to calculate the same answer.
- Always use a calculator wherever possible.
- Sketch a simple diagram and show which values are given and which values must be found.
- Always end your solution by using a summary or a closing statement depending on the type of problem you are working with.

Example 6

Solve ΔABC.



Solution:

Find the measures of $\angle A$, $\angle B$, and side b.

Solve for $\angle A$.

$$\sin A = \frac{11}{27}$$

$$\sin A = 0.4074$$

Enter	Display
C	0
.4074	0.4074
Inv sin	24.04161118

To the nearest degree, $\angle A = 24°$.

Solve for $\angle B$.

$$\cos B = \frac{11}{27}$$

$$\cos B = 0.4074$$

Enter	Display
C	0
.4074	0.4074
Inv cos	65.95838882

To the nearest degree, $\angle B = 66°$.

To calculate b, use one of the following.

- $\tan B = \frac{b}{11}$

$$\tan B = \frac{b}{11}$$

$$\tan 66° = \frac{b}{11}$$

$$b = 11 \times 2.2460$$

$$b = 24.706$$

$$b \doteq 25$$

- $\sin B = \frac{b}{27}$

$$\sin B = \frac{b}{27}$$

$$\sin 66° = \frac{b}{27}$$

$$b = 27 \times 0.9135$$

$$b \doteq 24.665$$

$$b \doteq 25$$

- $\tan A = \frac{11}{b}$

$$\tan A = \frac{11}{b}$$

$$\tan 24° = \frac{11}{b}$$

$$b = 11 \div 0.4452$$

$$b \doteq 24.708$$

$$b \doteq 25$$

- $\cos A = \frac{b}{27}$

$$\cos A = \frac{b}{27}$$

$$\cos 24° = \frac{b}{27}$$

$$b = 27 \times 0.9135$$

$$b = 24.665$$

$$b \doteq 25$$

Once $\angle A$ or $\angle B$ is found you could have used $\angle A + \angle B + \angle C = 180°$ to find the third angle.

$$\angle A + \angle B + \angle C = 180°$$

$$24° + \angle B + 90° = 180°$$

$$\angle B + 114° = 180°$$

$$\angle B = 66°$$

$$\angle A + \angle B + \angle C = 180°$$

$$\angle A + 66° + 90° = 180°$$

$$\angle A + 156° = 180°$$

$$\angle A = 24°$$

- the Theorem of Pythagoras

$$c^2 = a^2 + b^2$$
$$27^2 = 11^2 + b^2$$
$$729 = 121 + b^2$$
$$b^2 = 608$$
$$\sqrt{b^2} = \sqrt{608}$$
$$b \doteq 24.658$$
$$b \doteq 25$$

No matter which method is used, the result is the same.

Summary:

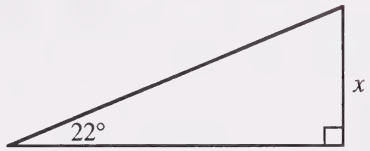
$\angle A = 24°$	$a = 11$
$\angle B = 66°$	$b = 25$
$\angle C = 90°$	$c = 27$

Example 7

A tree casts a shadow which is 12.3 m long. From the tip of the shadow, the angle of elevation to the top of the tree is 22°. What is the height of the tree? Round the answer to one decimal place.

Solution:

Sketch a diagram.



Find x.

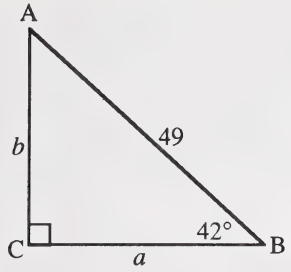
$$\tan 22° = \frac{x}{12.3}$$
$$0.4040 = \frac{x}{12.3}$$
$$x = 12.3 \times 0.4040$$
$$x \doteq 4.969$$
$$x \doteq 5.0$$

The height of the tree is approximately 5.0 m.

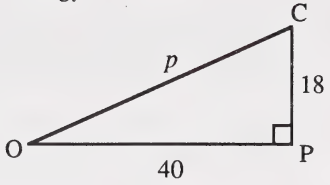
There must be a zero in 5.0 since the answer is rounded to one decimal place.

1. Solve each triangle. Round the answer to one decimal place.

a.



b.



2. The angle of depression of a raft on a lake is 18° from a cottage on a cliff that is 232 m above the water of the lake. Find the distance of the raft from the base of the cliff. Round the answer to the nearest metre.

3. The angle of elevation of a mountain road is 14°. Find the rise of the road for every 185 m measured along the roadway. Round the answer to the nearest metre.

4. A ski slope has a vertical drop of 210 m with a ski run that totals 750 m. Find the angle of elelvation of the ski slope. Round the answer to the nearest degree.

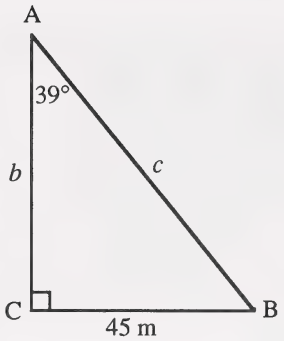
For solutions to **Extra Help**, turn to **Appendix A, Topic 4**.

Extensions

The following situations are common to problem solving when using the various trigonometric concepts.

- Several steps are needed before the required results are achieved.
- There are several ways to solve certain components of a right triangle.
- Once the side measures are found, it may be necessary to apply basic concepts to the triangle such as perimeter and area.

Example 8



Using ΔABC, do the following.

- Solve ΔABC. Round all the measures to the nearest whole number.

Solution:

The components needed are ∠B, side b, and side c.

$$\angle A + \angle B + \angle C = 180°$$
$$39° + \angle B + 90° = 180°$$
$$\angle B = 180° - 129°$$
$$\angle B = 51°$$

To find side b, use $\tan B = \frac{b}{45}$.

$$\tan 51° = \frac{b}{45}$$
$$1.2349 = \frac{b}{45}$$
$$b = 45 \times 1.2349$$
$$b = 55.5705$$
$$b \doteq 56$$

To find side c, use $\cos B = \frac{45}{c}$.

$$\cos 51° = \frac{45}{c}$$
$$c = 45 \div 0.6293$$
$$c \doteq 71.508$$
$$c \doteq 72$$

Summary:

∠A = 39°	a = 45 m
∠B = 51°	b = 56 m
∠C = 90°	c = 72 m

- Find the perimeter of ΔABC, and find the cost of fencing this triangle if fencing materials cost \$3.74 / m.

Solution:

$P = a + b + c$
$= 45 + 56 + 72$
$= 173$ m

The perimeter is 173 m.

Cost of fencing

$$C = 173 \times \$3.74$$
$$C = \$647.02$$

The cost of fencing ΔABC would be \$647.02.

- Find the area of ΔABC, and find the cost to sod this triangular lot if the materials cost \$17.46 / m^2.

Solution:

$$A = \frac{1}{2}ab$$
$$A = \frac{1}{2} \times 45 \times 56$$
$$A = \frac{1}{2} \times 2520$$
$$A = 1260$$

The area of the triangular lot is 1260 m^2.

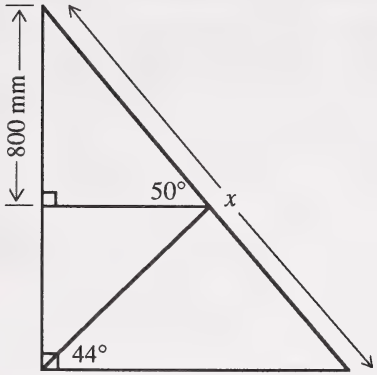
$$C = 1260 \times \$17.46$$
$$C = \$21\,999.60$$

The cost of sodding the triangular lot would be \$21 999.60.

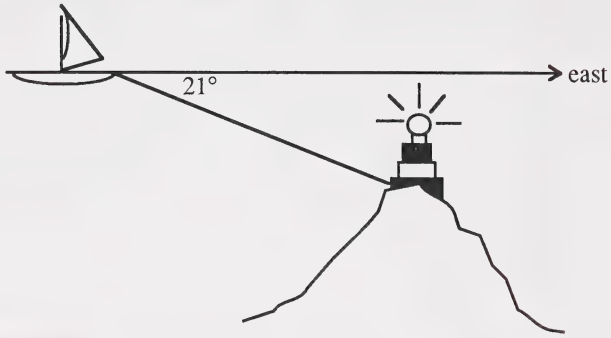
Try the following questions.

1. A tunnel is to be constructed through a hill. The angles of elevation to the top of the hill from points on opposite sides of the hill but on the same level are 27° and 42°. The top of the hill is 1650 m above all this. Find the length of the tunnel which will eventually join the two points on opposite sides of the hill. Round the answer to the nearest metre.

2. Solve for x. Express your answer to the nearest millimetre, centimetre, and metre.



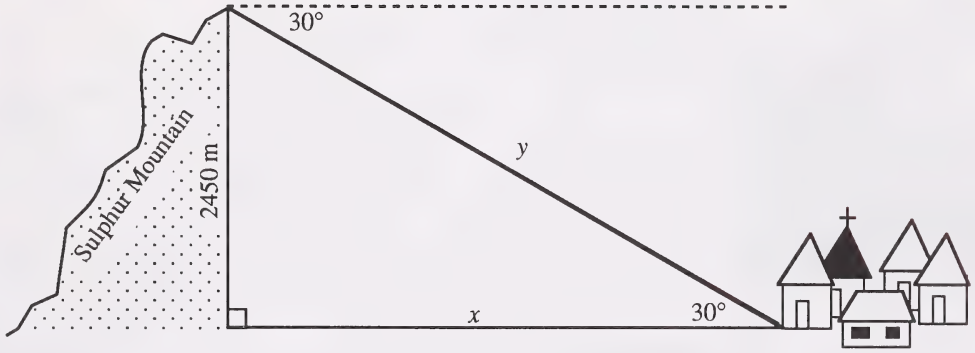
3. A ship is sailing in an easterly direction at a speed of 18.4 km/h. In 45 min, the ship is directly opposite the lighthouse. Find how close the ship comes to the shore and the lighthouse.



For solutions to **Extensions**, turn to **Appendix A**, **Topic 4**.

The final exercise in this unit is to solve the challenge which faced Colin and Jason when they went to the top of Sulphur Mountain in Banff National Park.

Start with a simple diagram.



Let the distance to the Banff townsite be x.
Let the distance from the top of the mountain be y.

Solve for x using the tangent ratio.

$$\tan 30° = \frac{2450}{x}$$

$$0.5774 = \frac{2450}{x}$$

$$x = \frac{2450}{0.5774}$$

$$x \doteq 4243.159$$

$$x \doteq 4243$$

The distance to the nearest metre from the bottom of Sulphur Mountain to the Banff townsite is 4243 m.

Solve for y by using the sine ratio.

$$\sin 30° = \frac{2450}{y}$$

$$0.5000 = \frac{2450}{y}$$

$$y = \frac{2450}{0.5000}$$

$$y = 4900$$

The distance from the townsite to the peak of Sulphur Mountain is 4900 m.

By using the cosine ratio, you would get the same result.

$$\cos 30° = \frac{4243}{y}$$

$$0.8660 = \frac{4243}{y}$$

$$y = \frac{4243}{0.8660}$$

$$y \doteq 4899.54$$

$$y \doteq 4900$$

The distance from the townsite to the peak of Sulphur Mountain is 4900 m.

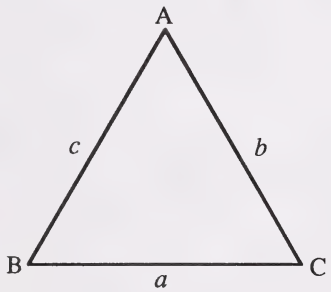
You can also use the cosine ratio to solve for y after the x-value is calculated.

Unit Summary

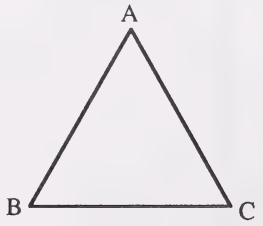
What You Have Learned

This unit on trigonometry has introduced you to the following concepts.

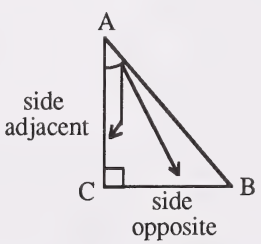
- A triangle is labelled using capital letters for the vertices and corresponding lower case letters for the sides.

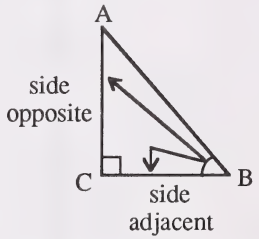


- The sum of the three angles of any triangle is 180°. In ΔABC, $\angle A + \angle B + \angle C = 180°$.



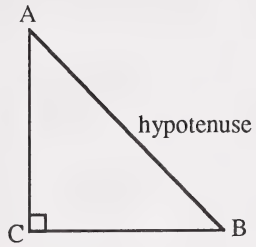
- In a right triangle, each acute angle or each reference angle has a side opposite it and a side adjacent to it.



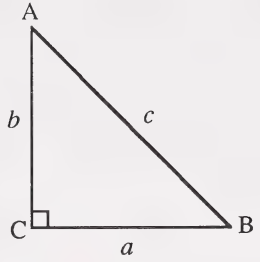


Unit Summary

- In a right triangle, the side opposite the right angle or the longest side of the three sides is called the hypotenuse.



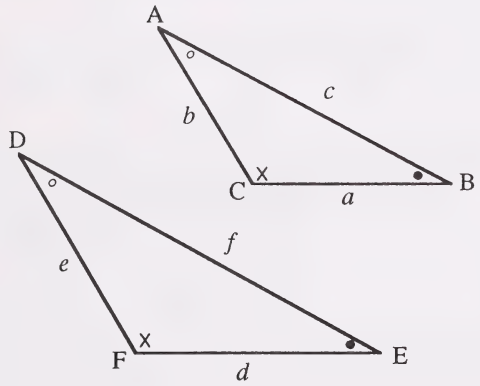
- When two sides of a right triangle are given, the third side can be calculated using the Theorem of Pythagoras.



In ΔABC, $c^2 = a^2 + b^2$.

Unit Summary

- Two triangles are similar if the corresponding angles are equal and if the corresponding sides are proportional.



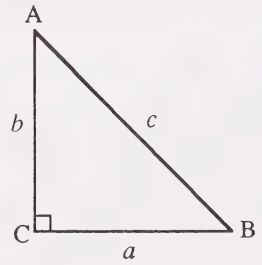
$\angle A = \angle D$, so $\angle A \leftrightarrow \angle D$.

$\angle B = \angle E$, so $\angle B \leftrightarrow \angle E$.

$\angle C = \angle F$, so $\angle C \leftrightarrow \angle F$.

Also, $\dfrac{AC}{DF} = \dfrac{AB}{DE} = \dfrac{BC}{EF}$.

- In right ΔABC,



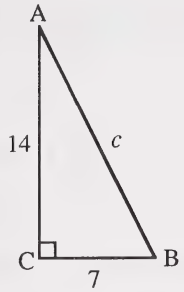
$\sin A = \dfrac{a}{c}$ $\quad \sin B = \dfrac{b}{c}$

$\cos A = \dfrac{b}{c}$ $\quad \cos B = \dfrac{a}{c}$

$\tan A = \dfrac{a}{b}$ $\quad \tan B = \dfrac{b}{a}$

Unit Summary

- When solving a right triangle, the unknown measures for the angles and the measures for the unknown sides are found.



In ΔABC, the unknown measures are $\angle A$, $\angle B$, and c.

You are now ready to complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Properties of Triangles

Topic 2 Similar Triangles

Topic 3 Developing the Trigonometric Ratios

Topic 4 Solving Right Triangles

Appendix B
Charts and Tables

Trigonometric Tables

Appendix A
Solutions

Review

1. a. ∠RST
   ∠TSR
   ∠S

   b. ∠XYZ
   ∠ZYX
   ∠Y

   c. ∠LMN
   ∠NML
   ∠M

2. a. 5° → acute ∠
   b. 179° → obtuse ∠
   c. 90° → right ∠
   d. 83° → acute ∠
   e. 98° → obtuse ∠
   f. 14° → acute ∠
   g. 125° → obtuse ∠

3. a. 27°
   b. 53°
   c. 19°

4. a. 68°
   b. 27°
   c. 84°

5. a. $\angle A = 180° - (63° + 42°)$
   $\angle A = 180° - 105°$
   $\angle A = 75°$

   b. $\angle ABC = 180° - 70°$
   $\angle ABC = 110°$

   $\angle A = 180° - (110° + 28°)$
   $\angle A = 180° - 138°$
   $\angle A = 42°$

   c. $\angle ABC = 180° - 140°$
   $\angle B = 40°$

   $\angle A = 180° - (40° + 90°)$
   $\angle A = 180° - 130°$
   $\angle A = 50°$

6. a. $\frac{2}{3} = \frac{n}{15}$
   $3n = 30$
   $n = 10$

   b. $\frac{n}{12} = \frac{3}{9}$
   $9n = 36$
   $n = 4$

   c. $\frac{12}{13} = \frac{36}{n}$
   $12n = 468$
   $n = 39$

   d. $\frac{5}{n} = \frac{15}{18}$
   $15n = 90$
   $n = 6$

e. $\frac{n}{6}=\frac{5}{25}$

$25n=30$

$n=1.2$ or $1\frac{1}{5}$

f. $\frac{3\frac{1}{2}}{n}=\frac{6\frac{1}{4}}{10\frac{1}{2}}$

$6\frac{1}{4}n=36\frac{3}{4}$

$n=5\frac{22}{25}$

g. $\frac{10.2}{12.6}=\frac{n}{4}$

$12.6n=40.8$

$n\doteq 3.24$

h. $\frac{5.25}{3\frac{1}{2}}=\frac{4.75}{n}$

$5.25n=16.625$

$n\doteq 3.17$

i. $\frac{n}{6}=\frac{9\frac{1}{3}}{14.5}$

$14.5n=56$

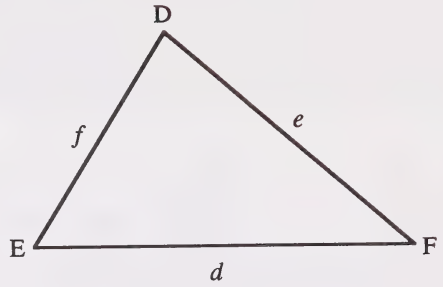
$n\doteq 3.86$

Exploring Topic 1

Activity 1

Label the vertices and sides of a triangle.

1. a.



ΔDEF or
ΔEDF or
ΔEFD or
ΔFDE or
ΔFED or
ΔDFE

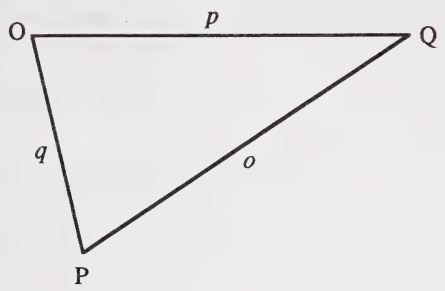
b. Angles:

- ∠D or ∠EDF or ∠FDE
- ∠E or ∠DEF or ∠FED
- ∠F or ∠DFE or ∠EFD

c. Sides:

- $\overline{DE}$ or $\overline{ED}$ or f
- $\overline{DF}$ or $\overline{FD}$ or e
- $\overline{EF}$ or $\overline{FE}$ or d

2. a.



ΔOPQ or
ΔOQP or
ΔPOQ or
ΔPQO or
ΔQOP or
ΔQPO

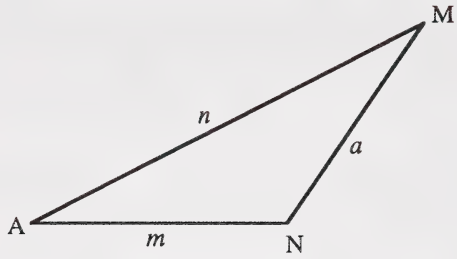
b. Angles:

- ∠O or ∠POQ or ∠QOP
- ∠P or ∠OPQ or ∠QPO
- ∠Q or ∠OQP or ∠PQO

c. Sides:

- $\overline{OP}$ or $\overline{PO}$ or q
- $\overline{PQ}$ or $\overline{QP}$ or o
- $\overline{QO}$ or $\overline{OQ}$ or p

3. a.



ΔAMN or
ΔANM or
ΔMAN or
ΔMNA or
ΔNAM or
ΔNMA

b. Angles:

- ∠M or ∠AMN or ∠NMA
- ∠A or ∠MAN or ∠NAM
- ∠N or ∠ANM or ∠MNA

c. Sides:

- $\overline{AM}$ or $\overline{MA}$ or n
- $\overline{MN}$ or $\overline{NM}$ or a
- $\overline{AN}$ or $\overline{NA}$ or m

Activity 2

Find the measure of the third angle of a triangle.

1. a. $\angle A + \angle B + \angle C = 180°$
$52° + 83° + \angle C = 180°$
$135° + \angle C = 180°$
$\angle C = 180° - 135°$
$\angle C = 45°$

b. $\angle A + \angle B + \angle C = 180°$
$34° + 23° + \angle C = 180°$
$57° + \angle C = 180°$
$\angle C = 180° - 57°$
$\angle C = 123°$

2. a. $\angle A + \angle B + \angle C = 180°$
$\angle A + 56° + 90° = 180°$
$\angle A + 146° = 180°$
$\angle A = 180° - 146°$
$\angle A = 34°$

b. $\angle A + \angle B + \angle C = 180°$
$39° + \angle B + 90° = 180°$
$129° + \angle B = 180°$
$\angle B = 180° - 129°$
$\angle B = 51°$

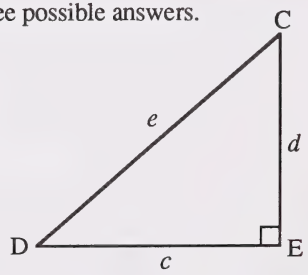
3. a. $\angle BAC = 180° - 95°$
$\angle BAC = 85°$
$\angle A + \angle B + \angle C = 180°$
$85° + \angle B + 75° = 180°$
$160° + \angle B = 180°$
$\angle B = 180° - 160°$
$\angle B = 20°$

b. $\angle ABC = 180° - 60°$
$\angle ABC = 120°$
$\angle A + \angle B + \angle C = 180°$
$43° + 120° + \angle C = 180°$
$163° + \angle C = 180°$
$\angle C = 180° - 163°$
$\angle C = 17°$

Activity 3

Label the sides of a right triangle as opposite, adjacent, and hypotenuse with respect to a given or chosen angle.

1. a. This is one of three possible answers.



b. For $\angle C$,

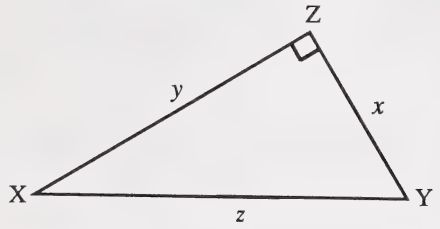
the side opposite is c or $\overline{DE}$
the side adjacent is d or $\overline{CE}$

For ∠D,

the side opposite is d or $\overline{CE}$
the side adjacent is c or $\overline{DE}$

c. The hypotenuse is e or $\overline{DC}$.

2. a. This is one of three possible answers.



b. For ∠X,

the side opposite is x or $\overline{YZ}$
the side adjacent is y or $\overline{XZ}$

For ∠Y,

the side opposite is y or $\overline{XZ}$
the side adjacent is x or $\overline{YZ}$

c. The hypotenuse is z or $\overline{XY}$.

Activity 4

Use the Theorem of Pythagoras.

1. $c^2 = a^2 + b^2$
$c^2 = 5^2 + 12^2$
$c^2 = 25 + 144$
$c^2 = 169$
$\sqrt{c^2} = \sqrt{169}$
$c = 13$
The value for c is 13.

2. $c^2 = a^2 + b^2$
$c^2 = 15^2 + 8^2$
$c^2 = 225 + 64$
$c^2 = 289$
$\sqrt{c^2} = \sqrt{289}$
$c = 17$
The value for c is 17.

3. $c^2 = a^2 + b^2$
$34^2 = a^2 + 30^2$
$1156 = a^2 + 900$
$a^2 = 1156 - 900$
$a^2 = 256$
$\sqrt{a^2} = \sqrt{256}$
$a = 16$
The value for a is 16.

4. $c^2 = a^2 + b^2$
$40^2 = a^2 + 24^2$
$1600 = a^2 + 576$
$a^2 = 1600 - 576$
$a^2 = 1024$
$\sqrt{a^2} = \sqrt{1024}$
$a = 32$
The value for a is 32.

5. $c^2 = a^2 + b^2$

$25^2 = 15^2 + b^2$

$625 = 225 + b^2$

$b^2 = 625 - 225$

$b^2 = 400$

$\sqrt{b^2} = \sqrt{400}$

$b = 20$

The value for b is 20.

6. $c^2 = a^2 + b^2$

$80^2 = 64^2 + b^2$

$6400 = 4096 + b^2$

$b^2 = 6400 - 4096$

$b^2 = 2304$

$\sqrt{b^2} = \sqrt{2304}$

$b = 48$

The value for b is 48.

7. a. $c^2 = a^2 + b^2$

$c^2 = 8^2 + 13^2$

$c^2 = 64 + 169$

$c^2 = 233$

$\sqrt{c^2} = \sqrt{233}$

$c \doteq 15.264$

$c \doteq 15$

To the nearest whole number, the value for c is 15.

b. $c^2 = a^2 + b^2$

$c^2 = 27^2 + 19^2$

$c^2 = 729 + 361$

$c^2 = 1090$

$\sqrt{c^2} = \sqrt{1090}$

$c \doteq 33.015$

$c \doteq 33$

To the nearest whole number, the value for c is 33.

8. a. $c^2 = a^2 + b^2$

$32^2 = a^2 + 14^2$

$1024 = a^2 + 196$

$a^2 = 1024 - 196$

$a^2 = 828$

$\sqrt{a^2} = \sqrt{828}$

$a \doteq 28.775$

$a \doteq 28.8$

To the nearest tenth, the value for a is 28.8.

b. $c^2 = a^2 + b^2$

$72^2 = a^2 + 31^2$

$5184 = a^2 + 961$

$a^2 = 5184 - 961$

$a^2 = 4223$

$\sqrt{a^2} = \sqrt{4223}$

$a \doteq 64.985$

$a \doteq 65.0$

To the nearest tenth, the value for a is 65.0.

9. a. $c^2 = a^2 + b^2$

$104^2 = 63^2 + b^2$

$10\,816 = 3969 + b^2$

$b^2 = 10\,816 - 3969$

$b^2 = 6847$

$b \doteq 82.747$

$b \doteq 82.75$

To the nearest hundredth, the value for b is 82.75.

b. $c^2 = a^2 + b^2$

$42^2 = 12^2 + b^2$

$1764 = 144 + b^2$

$b^2 = 1764 - 144$

$b^2 = 1620$

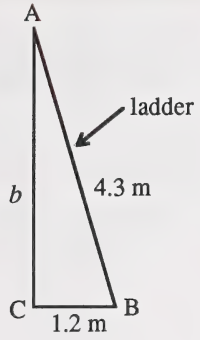
$\sqrt{b^2} = \sqrt{1620}$

$b \doteq 40.249$

$b \doteq 40.25$

To the nearest hundredth, the value for b is 40.25.

10.

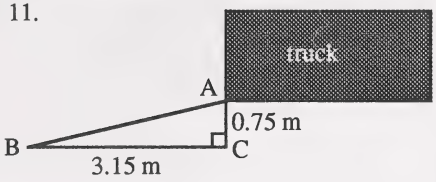


Solve for b.

$$c^2 = a^2 + b^2$$
$$(4.3)^2 = (1.2)^2 + b^2$$
$$18.49 = 1.44 + b^2$$
$$b^2 = 18.49 - 1.44$$
$$b^2 = 17.05$$
$$\sqrt{b^2} = \sqrt{17.05}$$
$$b \doteq 4.129$$
$$b \doteq 4.1$$

The ladder reaches to a point which is 4.1 m up the wall.

11.

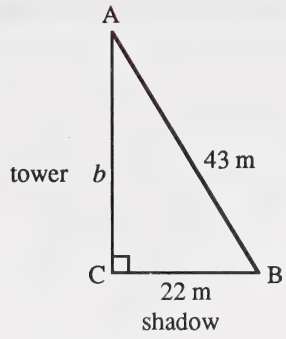


Solve for c.

$$c^2 = a^2 + b^2$$
$$c^2 = (3.15)^2 + (0.75)^2$$
$$c^2 = 9.9225 + 0.5625$$
$$c^2 = 10.485$$
$$\sqrt{c^2} = \sqrt{10.485}$$
$$c \doteq 3.238$$
$$c \doteq 3.24$$

The length of the ramp needed is 3.24 m.

12.



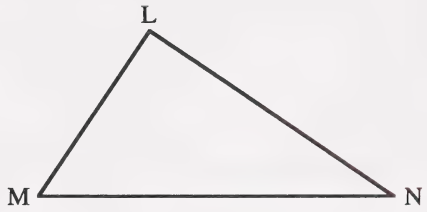
Solve for b.

$$c^2 = a^2 + b^2$$
$$43^2 = 22^2 + b^2$$
$$1849 = 484 + b^2$$
$$b^2 = 1365$$
$$\sqrt{b^2} = \sqrt{1365}$$
$$b \doteq 36.95$$
$$b \doteq 37$$

The height of the hydrotower is 37 m.

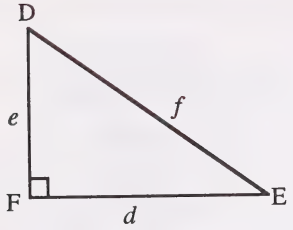
Extra Help

1. This is one possible solution.



2. a. For ∠D,
the side opposite is d
the side adjacent is e

For ∠E,
the side opposite is e
the side adjacent is d



b. The hypotenuse is f.

3. a. $\theta = 180° - 120°$
$\theta = 60°$

$\angle A + \angle B + \angle C = 180°$
$\angle A + 72° + 60° = 180°$
$\angle A + 132° = 180°$
$\angle A = 180° - 132°$
$\angle A = 48°$

Therefore, $\theta = 60°$ and $\alpha = 48°$.

b. $\theta = 180° - 130°$
$\theta = 50°$

$\angle A + \angle B + \angle C = 180°$
$50° + \angle B + 90° = 180°$
$\angle B + 140° = 180°$
$\angle B = 180° - 140°$
$\angle B = 40°$

Therefore, $\theta = 50°$ and $\alpha = 40°$.

4. a. $c^2 = a^2 + b^2$
$c^2 = (11.3)^2 + (17.7)^2$
$c^2 = 127.69 + 313.29$
$c^2 = 440.98$
$\sqrt{c^2} = \sqrt{440.98}$
$c \doteq 20.999\,523\,8$

i. 21
ii. 21.0
iii. 21.00
iv. 21.000

b. $c^2 = a^2 + b^2$
$24^2 = a^2 + 13^2$
$576 = a^2 + 169$
$a^2 = 576 - 169$
$a^2 = 407$
$\sqrt{a^2} = \sqrt{407}$
$a \doteq 20.174\,241$

i. 20
ii. 20.2
iii. 20.17
iv. 20.174

c. $c^2 = a^2 + b^2$
$(42.1)^2 = (16.9)^2 + b^2$
$1772.41 = 285.61 + b^2$
$b^2 = 1772.41 - 285.61$
$b^2 \doteq 1486.8$
$b = 38.559\,045\,63$

i. 39
ii. 38.6
iii. 38.56
iv. 38.559

5. $c^2 = a^2 + b^2$

$(507)^2 = (235)^2 + b^2$

$257\,049 = 55\,225 + b^2$

$b^2 = 257\,049 - 55\,225$

$b^2 = 201\,824$

$\sqrt{b^2} = \sqrt{201\,824}$

$b \doteq 449.248$

$b \doteq 449$

The missing length of the imagined triangle is 449 m.

Extensions

1. $(2x+7)° + (x+10)° + (2x+3)° = 180°$

$5x + 20 = 180$

$5x = 180 - 20$

$5x = 160$

$x = 32$

$\angle A = (2x + 7)°$
$= 2(32) + 7$
$= 64 + 7$
$= 71°$

$\angle B = (x + 10)°$
$= 32 + 10$
$= 42°$

$\angle C = (2x + 3)°$
$= 2(32) + 3$
$= 64 + 3$
$= 67°$

The answers are correct, since 71° + 42° + 67° = 180°.

2. a. $c^2 = a^2 + b^2$

$c^2 = \left(4\frac{1}{2}\right)^2 + \left(3\frac{3}{4}\right)^2$

$c^2 = 20.25 + 14.0625$

$c^2 = 34.3125$

$\sqrt{c^2} = \sqrt{34.3125}$

$c \doteq 5.8577$

$c \doteq 5.86$

The value for c is approximately 5.86.

b. $c^2 = a^2 + b^2$

$\left(12\frac{2}{5}\right)^2 = a^2 + \left(4\frac{3}{8}\right)^2$

$153.76 = a^2 + 19.140\,625$

$a^2 = 153.76 - 19.140\,625$

$a^2 = 134.619\,375$

$\sqrt{a^2} = \sqrt{134.619\,375}$

$a \doteq 11.602\,56$

$a \doteq 11.60$

The value for a is approximately 11.60.

c. $c^2 = a^2 + b^2$

$\left(16\frac{4}{5}\right)^2 = \left(3\frac{1}{8}\right)^2 + b^2$

$282.24 = 9.765\,625 + b^2$

$b^2 = 282.24 - 9.765\,625$

$b^2 = 272.474\,375$

$\sqrt{b^2} = \sqrt{272.474\,375}$

$b \doteq 16.5068$

$b \doteq 16.51$

The value for b is approximately 16.51.

The radical answer in each of these is exact, while the decimal answer is approximate.

3. a. $c^2 = a^2 + b^2$

$c^2 = \left(5\sqrt{2}\right)^2 + \left(3\sqrt{6}\right)^2$

$c^2 = 50 + 54$

$c^2 = 104$

$\sqrt{c^2} = \sqrt{104}$

$c = 2\sqrt{26}$

$c \doteq 2 \times 5.099$

$c \doteq 10.198$

$c \doteq 10.20$

The required values are $2\sqrt{26}$ and 10.20.

b. $c^2 = a^2 + b^2$

$\left(15\sqrt{2}\right)^2 = a^2 + \left(5\sqrt{6}\right)^2$

$450 = a^2 + 150$

$a^2 = 450 - 150$

$a^2 = 300$

$\sqrt{a^2} = \sqrt{300}$

$a = 10\sqrt{3}$

$a \doteq 10 \times 1.7321$

$a \doteq 17.321$

$a \doteq 17.32$

The required values are $10\sqrt{3}$ and 17.32.

c. $c^2 = a^2 + b^2$

$\left(12\sqrt{5}\right)^2 = \left(8\sqrt{3}\right)^2 + b^2$

$720 = 192 + b^2$

$b^2 = 720 - 192$

$b^2 = 528$

$\sqrt{b^2} = \sqrt{528}$

$b = 4\sqrt{33}$

$b \doteq 4 \times 5.7446$

$b \doteq 22.9784$

$b \doteq 22.98$

The required values are $4\sqrt{33}$ and 22. 98.

4. For ΔABC,

$$c^2 = a^2 + b^2$$
$$\left(20\sqrt{3}\right)^2 = a^2 + \left(10\sqrt{7}\right)^2$$
$$1200 = a^2 + 700$$
$$a^2 = 1200 - 700$$
$$a^2 = 500$$
$$\sqrt{a^2} = \sqrt{500}$$
$$a = 10\sqrt{5}$$
$$a \doteq 22.4 \text{ m}$$

For ΔBDE,

$$d^2 = b^2 + e^2$$
$$(25.6)^2 = (12.8)^2 + e^2$$
$$655.36 = 163.84 + e^2$$
$$e^2 = 655.36 - 163.84$$
$$e^2 = 491.52$$
$$\sqrt{e^2} = \sqrt{491.52}$$
$$e \doteq 22.17$$
$$e \doteq 22.2 \text{ m}$$

The distance between points C and D is
$a + e = 22.4 \text{ m} + 22.2 \text{ m}$
$= 44.6 \text{ m}.$

Exploring Topic 2

Activity 1

Define similar triangles and identify their properties.

1. If ΔABC ~ ΔMOP,

the angle relationships are
- ∠A ↔ ∠M or ∠BAC ↔ ∠OMP
- ∠B ↔ ∠O or ∠ABC ↔ ∠MOP
- ∠C ↔ ∠P or ∠ACB ↔ ∠MPO

the side relationships are
- $\overline{AB} \leftrightarrow \overline{MO}$ or $c \leftrightarrow p$
- $\overline{BC} \leftrightarrow \overline{OP}$ or $a \leftrightarrow m$
- $\overline{AC} \leftrightarrow \overline{MP}$ or $b \leftrightarrow o$

2. If ΔRED ~ ΔPOT,

the angle relationships are
- ∠R ↔ ∠P or ∠ERD ↔ ∠OPT
- ∠E ↔ ∠O or ∠RED ↔ ∠POT
- ∠D ↔ ∠T or ∠RDE ↔ ∠PTO

the side relationships are
- $\overline{RE} \leftrightarrow \overline{PO}$ or $d \leftrightarrow t$
- $\overline{ED} \leftrightarrow \overline{OT}$ or $r \leftrightarrow p$
- $\overline{RD} \leftrightarrow \overline{PT}$ or $e \leftrightarrow o$

3. ΔCAT ~ ΔPON

Angles:	Sides:
∠C ↔ ∠P	$\overline{CA} \leftrightarrow \overline{PO}$
∠A ↔ ∠O	$\overline{AT} \leftrightarrow \overline{ON}$
∠T ↔ ∠N	$\overline{CT} \leftrightarrow \overline{PN}$

4. $\Delta WXY \sim \Delta DOT$

Angles:	Sides:
$\angle W \leftrightarrow \angle D$	$\overline{WX} \leftrightarrow \overline{DO}$
$\angle X \leftrightarrow \angle O$	$\overline{XY} \leftrightarrow \overline{OT}$
$\angle Y \leftrightarrow \angle T$	$\overline{WY} \leftrightarrow \overline{DT}$

Activity 2

Use the ratio property of similar triangles to find the lengths of corresponding sides.

1. a. Since $\Delta RAT \sim \Delta MOP$,

in ΔRAT,	in ΔMOP,
$\angle R = 31°$	$\angle O = 114°$
$\angle T = 180° - (31° + 114°)$	$\angle P = 35°$
$= 180° - 145°$	
$= 35°$	

b.

$\angle R \leftrightarrow \angle M$	$t \leftrightarrow p$
$\angle A \leftrightarrow \angle O$	$r \leftrightarrow m$
$\angle T \leftrightarrow \angle P$	$a \leftrightarrow o$

2. a. Since $\Delta ABC \sim \Delta TPO$,

in ΔABC,	in ΔPOT,
$\angle A = 30°$	$\angle P = 60°$
$\angle B = 180° - (90° + 30°)$	
$= 180° - 120°$	
$= 60°$	

b. The similarity relationship for the angles is

$\angle A \leftrightarrow \angle T$

$\angle B \leftrightarrow \angle P$

$\angle C \leftrightarrow \angle O$

The similarity relationship for the sides is

$c \leftrightarrow o$

$a \leftrightarrow t$

$b \leftrightarrow p$

3. a. ΔABC is not similar to ΔDEF.

4. a. ΔABC is not similar to ΔDEF.

5. a. $\Delta MON \sim \Delta POQ$

b.

Angles:	Sides:
$\angle M \leftrightarrow \angle P$	$x \leftrightarrow y$
$\angle O \leftrightarrow \angle O$	$m \leftrightarrow p$
$\angle N \leftrightarrow \angle Q$	$n \leftrightarrow q$

6. a. $\Delta SUT \sim \Delta XZY$

b. Angles:

$\angle S \leftrightarrow \angle X$

$\angle U \leftrightarrow \angle Z$

$\angle T \leftrightarrow \angle Y$

Sides:

$s \leftrightarrow x$

$u \leftrightarrow z$

$t \leftrightarrow y$

7. a. $\Delta MON \sim \Delta PRQ$

Angles:

$\angle M = \angle P = 87°$

$\angle R = \angle O = 42°$

$\angle Q = 180° - (87° + 42°)$

$= 180° - 129°$

$= 51°$

$\angle Q = \angle N = 51°$

Sides:

$\frac{o}{r} = \frac{n}{q} = \frac{m}{p}$

$\frac{o}{r} = \frac{24}{12} = \frac{m}{15}$

$\frac{24}{12} = \frac{m}{15}$

$12m = 24 \times 15$

$12m = 360$

$m = 30$

$\frac{o}{r}$ can be any two values which simplify to $\frac{2}{1}$.

For example, $\frac{2}{1}, \frac{4}{2}, \frac{6}{3}, \frac{8}{4}, \frac{10}{5}, \frac{12}{6}, \frac{14}{7}, \frac{16}{8}, \frac{18}{9}, \ldots$.

The most logical one for o is 18, and the one for r is 9.

Use $c^2 = a^2 + b^2$ to confirm this. (Even though this is not a right triangle, 87° is near to 90° so the equation will give a close approximation of the answer.)

b. $\Delta STU \sim \Delta ZYW$

Angles:

$\angle S = 180° - (67° + 65°)$

$= 180° - 132°$

$= 48°$

$\angle S = \angle Z = 48°$

$\angle T = \angle Y = 67°$

Sides:

$\frac{t}{y} = \frac{s}{z} = \frac{u}{w}$

$\frac{4.3}{1.3} = \frac{3.9}{z} = \frac{4.2}{w}$

Solve for z.

$\frac{4.3}{1.3} = \frac{3.9}{z}$

$4.3z = 1.3 \times 3.9$

$4.3z = 5.07$

$z = \frac{5.07}{4.3}$

$z \doteq 1.179$

$z \doteq 1.2$

Solve for w.

$\frac{4.3}{1.3} = \frac{4.2}{w}$

$4.3w = 4.2 \times 1.3$

$4.3w = 5.46$

$w = \frac{5.46}{4.3}$

$w \doteq 1.2698$

$w \doteq 1.3$

c. $\Delta RAT \sim \Delta OBY$

Angles:

$\angle B = \angle A = 90°$

$\angle T = \angle Y = 40°$

$\angle O = 180° - (90° + 40°)$

$= 180° - 130°$

$= 50°$

$\angle O = \angle R = 50°$

Sides:

$$\frac{a}{b} = \frac{t}{y} = \frac{r}{o}$$

$$\frac{13.7}{9.8} = \frac{10.5}{o}$$

$$13.7o = 10.5 \times 9.8$$

$$13.7o = 102.9$$

$$o = \frac{102.9}{13.7}$$

$$o \doteq 7.5109$$

$$o \doteq 7.5$$

To find r, use

$$a^2 = t^2 + r^2.$$

$$(13.7)^2 = t^2 + (10.5)^2$$

$$187.69 = t^2 + 110.25$$

$$t^2 = 187.69 - 110.25$$

$$t^2 = 77.44$$

$$\sqrt{t^2} = \sqrt{77.44}$$

$$t = 8.8$$

$$\frac{13.7}{9.8} = \frac{8.8}{y}$$

$$13.7y = 8.8 \times 9.8$$

$$13.7y = 86.24$$

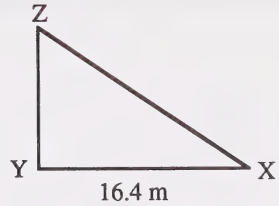
$$y = \frac{86.24}{13.7}$$

$$y \doteq 6.294$$

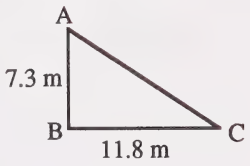
$$y \doteq 6.3$$

8. a. $\frac{p}{g} = \frac{r}{\boxed{k}}$ b. $\frac{s}{\boxed{p}} = \frac{h}{g}$ c. $\frac{r}{s} = \frac{\boxed{k}}{h}$ d. $\frac{\boxed{h}}{s} = \frac{g}{p}$

9. shed



pole



$$\frac{ZY}{AB} = \frac{XY}{CB}$$

$$\frac{ZY}{7.3} = \frac{16.4}{11.8}$$

$$11.8ZY = 16.4 \times 7.3$$

$$11.8ZY = 119.72$$

$$ZY = \frac{119.72}{11.8}$$

$$ZY \doteq 10.146$$

$$ZY \doteq 10.1$$

The height of the shed is 10.1 m.

10. $\frac{x}{16.90} = \frac{35.40}{12.30}$

$$12.30x = 35.40 \times 16.90$$

$$12.30x = 598.26$$

$$x = \frac{598.26}{12.30}$$

$$x \doteq 48.639$$

$$x \doteq 48.64$$

To the nearest hundredth, the width of the river is 48.64 m.

Extra Help

1. $\angle M \leftrightarrow \angle R$ $\quad m \leftrightarrow r$
   $\angle O \leftrightarrow \angle A$ $\quad o \leftrightarrow a$
   $\angle P \leftrightarrow \angle T$ $\quad p \leftrightarrow t$

2. $\angle P \leftrightarrow \angle M$ $\quad p \leftrightarrow m$
   $\angle O \leftrightarrow \angle A$ $\quad o \leftrightarrow a$
   $\angle T \leftrightarrow \angle N$ $\quad t \leftrightarrow n$

3. $\dfrac{3.7}{t} = \dfrac{7.3}{3.4} = \dfrac{9.6}{a}$

   $\dfrac{3.7}{t} = \dfrac{7.3}{3.4}$

   $7.3t = 3.7 \times 3.4$

   $7.3t = 12.58$

   $t = \dfrac{12.58}{7.3}$

   $t \doteq 1.723$

   $t \doteq 1.7$

   $\dfrac{7.3}{3.4} = \dfrac{9.6}{a}$

   $7.3a = 9.6 \times 3.4$

   $7.3a = 32.64$

   $a = \dfrac{32.64}{7.3}$

   $a \doteq 4.471$

   $a \doteq 4.5$

   $\angle A = \angle O = 118°$

   $\angle R = \angle L = 42°$

   $\angle P = 180° - (118° + 42°)$

   $= 180° - 160°$

   $= 20°$

   $\angle T = \angle P = 20°$

4. a. $\dfrac{b}{t} = \dfrac{d}{p} = \dfrac{o}{a}$

   $\dfrac{b}{6.5} = \dfrac{9.3}{p} = \dfrac{14.6}{11.4}$

   $\dfrac{b}{6.5} = \dfrac{14.6}{11.4}$

   $11.4b = 6.5 \times 14.6$

   $11.4b = 94.9$

   $b = \dfrac{94.9}{11.4}$

   $b \doteq 8.3$

   $\dfrac{9.3}{p} = \dfrac{14.6}{11.4}$

   $14.6p = 9.3 \times 11.4$

   $14.6p = 106.02$

   $p = \dfrac{106.02}{14.6}$

   $p \doteq 7.3$

   b. Check for b by using

   $\dfrac{b}{o} = \dfrac{t}{a}$.

   $\dfrac{b}{14.6} = \dfrac{6.5}{11.4}$

   $11.4b = 14.6 \times 6.5$

   $b \doteq 8.3$ (same answer as before)

   Check for p by using

   $\dfrac{o}{d} = \dfrac{a}{p}$.

   $\dfrac{14.6}{9.3} = \dfrac{11.4}{p}$

   $14.6p = 9.3 \times 11.4$

   $p \doteq 7.3$ (same answer as before)

Extensions

1. $$\frac{54}{72}=\frac{45}{45+x}$$
$$54(45+x)=72\times 45$$
$$2430+54x=3240$$
$$54x=3240-2430$$
$$54x=810$$
$$x=\frac{810}{54}$$
$$x=15$$

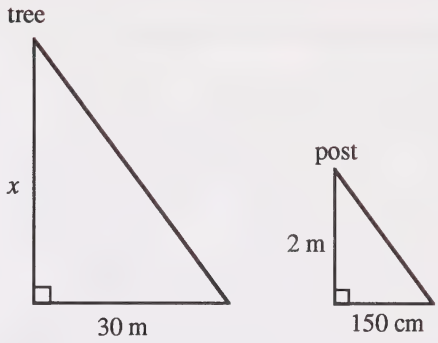
The value for x is 15.

2. $$\frac{y}{y-10.8}=\frac{32.2}{18.6}$$
$$32.2(y-10.8)=18.6y$$
$$32.2y-347.76=18.6y$$
$$32.2y-18.6y=347.76$$
$$13.6y=347.76$$
$$y=\frac{347.76}{13.6}$$
$$y\doteq 25.6$$

The value for y is 25.6.

3. Let $\overline{DA}$ be x.
$$\frac{48+x}{x}=\frac{80}{64}$$
$$64(48+x)=80x$$
$$3072+64x=80x$$
$$80x-64x=3072$$
$$16x=3072$$
$$x=\frac{3072}{16}$$
$$x=192$$

The distance across the river is 192 m.

4. 

Let x be the height of the tree.

- Height in metres

$$150\text{ cm}=150\text{ cm}\times\frac{1\text{ m}}{100\text{ cm}}$$
$$=1.5\text{ m}$$

$$\frac{x}{2}=\frac{30}{1.5}$$
$$1.5x=2\times 30$$
$$1.5x=60$$
$$x=\frac{60}{1.5}$$
$$x=40$$

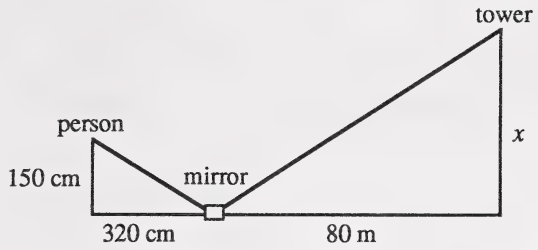
The height of the tree is 40 m.

- Height in centimetres

$$\frac{x}{200} = \frac{3000}{150}$$
$$150x = 200 \times 3000$$
$$150x = 600\,000$$
$$x = \frac{600\,000}{150}$$
$$x = 4000$$

The height of the tree, in centimetres, is 4000 cm.

5.



Let x be the height of the tower.

- Height in metres

$$\frac{1.5}{x} = \frac{3.2}{80}$$
$$3.2x = 1.5 \times 80$$
$$3.2x = 120$$
$$x = \frac{120}{3.2}$$
$$x = 37.5$$

The height of the tower is 37.5 m.

- Height in centimetres

$$\frac{150}{x} = \frac{320}{8000}$$
$$320x = 150 \times 8000$$
$$320x = 1\,200\,000$$
$$x = \frac{1\,200\,000}{320}$$
$$x = 3750$$

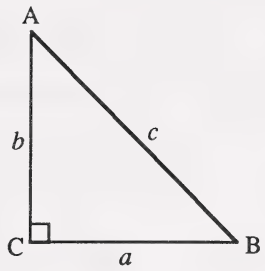
The height of the tower is 3750 cm.

Exploring Topic 3

Activity 1

Define the tangent, sine, and cosine ratios.

1. a.



For angle A,

$\sin A = \frac{a}{c}$

$\cos A = \frac{b}{c}$

$\tan A = \frac{a}{b}$

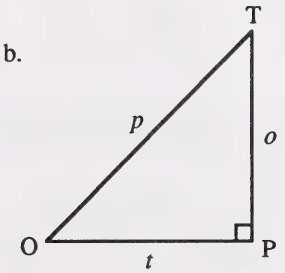
For angle B,

$\sin B = \frac{b}{c}$

$\cos B = \frac{a}{c}$

$\tan B = \frac{b}{a}$

b.



For angle T,

$\sin T = \frac{t}{p}$

$\cos T = \frac{o}{p}$

$\tan T = \frac{t}{o}$

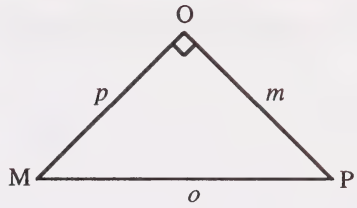
For angle O,

$\sin O = \frac{o}{p}$

$\cos O = \frac{t}{p}$

$\tan O = \frac{o}{t}$

2. a.



For angle M,

$\sin M = \frac{m}{o}$

$\cos M = \frac{p}{o}$

$\tan M = \frac{m}{p}$

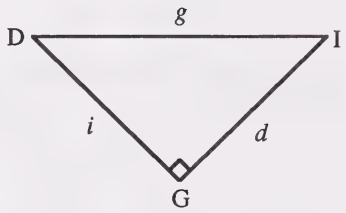
For angle P,

$\sin P = \frac{p}{o}$

$\cos P = \frac{m}{o}$

$\tan P = \frac{p}{m}$

b.



For angle D,

$\sin D = \frac{d}{g}$

$\cos D = \frac{i}{g}$

$\tan D = \frac{d}{i}$

For angle I,

$\sin I = \frac{i}{g}$

$\cos I = \frac{d}{g}$

$\tan I = \frac{i}{d}$

Activity 2

Determine the tangent, sine, and cosine ratios of any given acute angle.

1. a. Using $\angle A$ or θ,

$$\sin\theta = \frac{60}{75} = \frac{4}{5} = 0.8000$$

$$\cos\theta = \frac{45}{75} = \frac{3}{5} = 0.6000$$

$$\tan\theta = \frac{60}{45} = \frac{4}{3} \doteq 1.3333$$

Using $\angle B$ or α,

$$\sin\alpha = \frac{45}{75} = \frac{3}{5} = 0.6000$$

$$\cos\alpha = \frac{60}{75} = \frac{4}{5} = 0.8000$$

$$\tan\alpha = \frac{45}{60} = \frac{3}{4} = 0.7500$$

b. Using $\angle D$ or θ,

$$\sin\theta = \frac{42}{45} = \frac{14}{15} \doteq 0.9333$$

$$\cos\theta = \frac{17}{45} \doteq 0.3778$$

$$\tan\theta = \frac{42}{17} \doteq 2.4706$$

Using $\angle E$ or α,

$$\sin\alpha = \frac{17}{45} \doteq 0.3778$$

$$\cos\alpha = \frac{42}{45} = \frac{14}{15} \doteq 0.9333$$

$$\tan\alpha = \frac{17}{42} \doteq 0.4048$$

2. a. Using $\angle A$ or θ,

$$\sin\theta = \frac{46}{54} = \frac{23}{27} \doteq 0.8519$$

$$\cos\theta = \frac{28}{54} = \frac{14}{27} \doteq 0.5185$$

$$\tan\theta = \frac{46}{28} = \frac{23}{14} \doteq 1.6429$$

Using $\angle B$ or α,

$$\sin\alpha = \frac{28}{54} = \frac{14}{27} \doteq 0.5185$$

$$\cos\alpha = \frac{46}{54} = \frac{23}{27} \doteq 0.8519$$

$$\tan\alpha = \frac{28}{46} = \frac{14}{23} \doteq 0.6087$$

b. Using $\angle D$ or θ,

$$\sin\theta = \frac{64}{80} = \frac{4}{5} = 0.8000$$

$$\cos\theta = \frac{48}{80} = \frac{3}{5} = 0.6000$$

$$\tan\theta = \frac{64}{48} = \frac{4}{3} \doteq 1.3333$$

Using $\angle E$ or α,

$$\sin\alpha = \frac{48}{80} = \frac{3}{5} = 0.6000$$

$$\cos\alpha = \frac{64}{80} = \frac{4}{5} = 0.8000$$

$$\tan\alpha = \frac{48}{64} = \frac{3}{4} = 0.7500$$

3. a. $\sin 39° = 0.6293$
   $\cos 39° = 0.7771$
   $\tan 39° = 0.8098$

   b. $\sin 63° = 0.8910$
   $\cos 63° = 0.4540$
   $\tan 63° = 1.9626$

   c. $\sin 23° = 0.3907$
   $\cos 23° = 0.9205$
   $\tan 23° = 0.4245$

4. a. $\sin 14° = 0.2419$
   $\cos 14° = 0.9703$
   $\tan 14° = 0.2493$

   b. $\sin 53° = 0.7986$
   $\cos 53° = 0.6018$
   $\tan 53° = 1.3270$

   c. $\sin 89° = 0.9998$
   $\cos 89° = 0.0175$
   $\tan 89° = 57.2900$

Activity 3

Determine the measure of an acute angle given the tangent, sine, or cosine ratio.

1. a.

Enter	Display
C	0
2.3006	2.3006
Inv tan	66.50689854

To the nearest degree, $\angle B = 67°$.

b.

Enter	Display
C	0
.0659	0.0659
Inv cos	86.22146985

To the nearest degree,
$\angle B = 86°$.

c.

Enter	Display
C	0
.5342	0.5342
Inv sin	32.28967364

To the nearest degree,
$\angle B = 32°$.

d.

Enter	Display
C	0
.3542	0.3542
Inv tan	19.50414549

To the nearest degree,
$\angle B = 20°$.

e.

Enter	Display
C	0
.7235	0.7235
Inv cos	43.65579186

To the nearest degree,
$\angle B = 44°$.

f.

Enter	Display
C	0
.8993	0.8993
Inv sin	64.06620737

To the nearest degree,
$\angle B = 64°$.

2. a.

Enter	Display
C	0
.9644	0.9644
Inv sin	74.66585631

To the nearest degree,
$\angle C = 75°$.

b.

Enter	Display
C	0
1.4439	1.4439
Inv tan	55.29473692

To the nearest degree,
$\angle B = 55°$.

c.

Enter	Display
C	0
.5336	0.5336
Inv tan	28.08438094

To the nearest degree,
$\angle B = 28°$.

d.

Enter	Display
C	0
.3470	0.347
Inv sin	20.3039311

To the nearest degree,
$\angle B = 20°$.

e.

Enter	Display
C	0
.1142	0.1142
Inv cos	83.44251556

To the nearest degree,
$\angle B = 83°$.

f.

Enter	Display
C	0
.7946	0.7946
Inv cos	37.38250905

To the nearest degree,
$\angle B = 37°$.

3. a. 0.2374 is between 0.2250 and 0.2419 or between 76° and 77°.

Differences: 0.2374 – 0.2250 = 0.0124
0.2419 – 0.2374 = 0.0045
0.2374 is closer to 0.2419, so ∠C = 76°.

b. 3.4009 is between 3.2709 and 3.4874 or between 73° and 74°.

Differences: 3.4009 – 3.2709 = 0.1300
3.4874 – 3.4009 = 0.0865
3.4009 is closer to 3.4874, so ∠C = 74°.

c. 0.7934 is between 0.7880 and 0.7986 or between 52° and 53°.

Differences: 0.7934 – 0.7880 = 0.0054
0.7986 – 0.7934 = 0.0052
0.7934 is closer to 0.7986, so ∠C = 53°.

4. a. 0.2084 is between 0.2079 and 0.2250 or between 12° and 13°.

Differences: 0.2084 – 0.2079 = 0.0005
0.2250 – 0.2084 = 0.0166
0.2084 is closer to 0.2079, so ∠D = 12°.

b. 0.9246 is between 0.9205 and 0.9272 or between 22° and 23°.

Differences: 0.9246 – 0.9205 = 0.0041
0.9272 – 0.9246 = 0.0026
0.9246 is closer to 0.9272, so ∠D = 22°.

c. 0.5539 is between 0.5317 and 0.5543 or between 28° and 29°.

Differences: 0.5539 – 0.5317 = 0.0222
0.5543 – 0.5539 = 0.0004
0.5539 is closer to 0.5543, so ∠D = 29°.

Activity 4

Determine the tangent, sine, or cosine ratios given two sides of a right triangle.

1. a. $c^2 = a^2 + b^2$

$c^2 = 5^2 + 12^2$

$c^2 = 25 + 144$

$c^2 = 169$

$\sqrt{c^2} = \sqrt{169}$

$c = 13$

For $\angle A$,

$$\sin A = \frac{5}{13}$$
$$\doteq 0.3846$$
$$\cos A = \frac{12}{13}$$
$$\doteq 0.9231$$
$$\tan A = \frac{5}{12}$$
$$\doteq 0.4167$$

For $\angle B$,

$$\sin B = \frac{12}{13}$$
$$\doteq 0.9231$$
$$\cos B = \frac{5}{13}$$
$$\doteq 0.3846$$
$$\tan B = \frac{12}{5}$$
$$= 2.4000$$

b. $f^2 = e^2 - d^2$

$f^2 = 24^2 - 9^2$

$f^2 = 576 - 81$

$f^2 = 495$

$\sqrt{f^2} = \sqrt{495}$

$f \doteq 22$

For $\angle D$,

$$\sin D = \frac{9}{24}$$
$$= \frac{3}{8}$$
$$= 0.3750$$

For $\angle F$,

$$\sin F = \frac{22}{24}$$
$$= \frac{11}{12}$$
$$\doteq 0.9167$$

$$\cos D = \frac{22}{24}$$
$$= \frac{11}{12}$$
$$\doteq 0.9167$$
$$\tan D = \frac{9}{22}$$
$$\doteq 0.4091$$

$$\cos F = \frac{9}{24}$$
$$= \frac{3}{8}$$
$$= 0.3750$$
$$\tan F = \frac{22}{9}$$
$$\doteq 2.4444$$

c. $g^2 = h^2 - i^2$

$g^2 = 40^2 - 24^2$

$g^2 = 1600 - 576$

$g^2 = 1024$

$\sqrt{g^2} = \sqrt{1024}$

$g = 32$

For $\angle G$,

$$\sin G = \frac{32}{40}$$
$$= \frac{4}{5}$$
$$= 0.8000$$
$$\cos G = \frac{24}{40}$$
$$= \frac{3}{5}$$
$$= 0.6000$$

For $\angle I$,

$$\sin I = \frac{24}{40}$$
$$= \frac{3}{5}$$
$$= 0.6000$$
$$\cos I = \frac{32}{40}$$
$$= \frac{4}{5}$$
$$= 0.8000$$

$\tan G = \frac{32}{24}$

$= \frac{4}{3}$

$\doteq 1.3333$

$\tan I = \frac{24}{32}$

$= \frac{3}{4}$

$= 0.7500$

$\tan M = \frac{12}{16}$

$= \frac{3}{4}$

$= 0.7500$

$\tan P = \frac{16}{12}$

$= \frac{4}{3}$

$\doteq 1.3333$

d. $o^2 = m^2 + p^2$

$o^2 = 12^2 + 16^2$

$o^2 = 144 + 256$

$o^2 = 400$

$\sqrt{o^2} = \sqrt{400}$

$o = 20$

For $\angle M$,

$\sin M = \frac{12}{20}$

$= \frac{3}{5}$

$= 0.6000$

$\cos M = \frac{16}{20}$

$= \frac{4}{5}$

$= 0.8000$

For $\angle P$,

$\sin P = \frac{16}{20}$

$= \frac{4}{5}$

$= 0.8000$

$\cos P = \frac{12}{20}$

$= \frac{3}{5}$

$= 0.6000$

e. $a^2 = t^2 - r^2$

$a^2 = 52^2 - 16^2$

$a^2 = 2704 - 256$

$a^2 = 2448$

$\sqrt{a^2} = \sqrt{2448}$

$a \doteq 49$

For $\angle R$,

$\sin R = \frac{16}{52}$

$= \frac{4}{13}$

$\doteq 0.3077$

$\cos R = \frac{49}{52}$

$\doteq 0.9423$

$\tan R = \frac{16}{49}$

$\doteq 0.3265$

For $\angle A$,

$\sin A = \frac{49}{52}$

$\doteq 0.9423$

$\cos A = \frac{16}{52}$

$= \frac{4}{13}$

$\doteq 0.3077$

$\tan A = \frac{49}{16}$

$= 3.0625$

f. $m^2 = n^2 - e^2$

$m^2 = 80^2 - 35^2$

$m^2 = 6400 - 1225$

$m^2 = 5175$

$\sqrt{m^2} = \sqrt{5175}$

$m \doteq 72$

For $\angle$M,

$\sin M = \frac{72}{80}$

$= \frac{9}{10}$

$= 0.9000$

$\cos M = \frac{35}{80}$

$= \frac{7}{16}$

$= 0.4375$

$\tan M = \frac{72}{35}$

$\doteq 2.0571$

For $\angle$E,

$\sin E = \frac{35}{80}$

$= \frac{7}{16}$

$= 0.4375$

$\cos E = \frac{72}{80}$

$= \frac{9}{10}$

$= 0.9000$

$\tan E = \frac{35}{72}$

$\doteq 0.4861$

g. $e^2 = t^2 + m^2$

$e^2 = 20^2 + 15^2$

$e^2 = 400 + 225$

$e^2 = 625$

$\sqrt{e^2} = \sqrt{625}$

$e = 25$

For $\angle$M,

$\sin M = \frac{15}{25}$

$= \frac{3}{5}$

$= 0.6000$

$\cos M = \frac{20}{25}$

$= \frac{4}{5}$

$= 0.8000$

$\tan M = \frac{15}{20}$

$= \frac{3}{4}$

$= 0.7500$

For $\angle$T,

$\sin T = \frac{20}{25}$

$= \frac{4}{5}$

$= 0.8000$

$\cos T = \frac{15}{25}$

$= \frac{3}{5}$

$= 0.6000$

$\tan T = \frac{20}{15}$

$= \frac{4}{3}$

$\doteq 1.3333$

h. $o^2 = b^2 - r^2$

$o^2 = 36^2 - 21^2$

$o^2 = 1296 - 441$

$o^2 = 855$

$\sqrt{o^2} = \sqrt{855}$

$o \doteq 29$

i. $c^2 = t^2 - a^2$

$c^2 = 84^2 - 35^2$

$c^2 = 7056 - 1225$

$c^2 = 5831$

$\sqrt{c^2} = \sqrt{5831}$

$c \doteq 76$

For $\angle$R,

$\sin R = \frac{21}{36}$

$= \frac{7}{12}$

$\doteq 0.5833$

$\cos R = \frac{29}{36}$

$\doteq 0.8056$

$\tan R = \frac{21}{29}$

$\doteq 0.7241$

For $\angle$T,

$\sin O = \frac{29}{36}$

$\doteq 0.8056$

$\cos O = \frac{21}{36}$

$= \frac{7}{12}$

$\doteq 0.5833$

$\tan O = \frac{29}{21}$

$\doteq 1.3810$

For $\angle$C,

$\sin C = \frac{76}{84}$

$= \frac{19}{21}$

$\doteq 0.9048$

$\cos C = \frac{35}{84}$

$= \frac{5}{12}$

$\doteq 0.4167$

$\tan C = \frac{76}{35}$

$\doteq 2.1714$

For $\angle$A,

$\sin A = \frac{35}{84}$

$= \frac{5}{12}$

$\doteq 0.4167$

$\cos A = \frac{76}{84}$

$\doteq 0.9048$

$\tan A = \frac{35}{76}$

$\doteq 0.4605$

Extra Help

1. a. For $\angle A$,

$$\sin A = \frac{16}{29}$$

$$\cos A = \frac{24}{29}$$

$$\tan A = \frac{16}{24} = \frac{2}{3}$$

For $\angle B$,

$$\sin B = \frac{24}{29}$$

$$\cos B = \frac{16}{29}$$

$$\tan B = \frac{24}{16} = \frac{3}{2}$$

b. For $\angle D$,

$$\sin D = \frac{50}{61}$$

$$\cos D = \frac{35}{61}$$

$$\tan D = \frac{50}{35} = \frac{10}{7}$$

For $\angle E$,

$$\sin E = \frac{35}{61}$$

$$\cos E = \frac{50}{61}$$

$$\tan E = \frac{35}{50} = \frac{7}{10}$$

2. a. For $\angle R$,

$$\sin R = \frac{12}{18} \doteq 0.6667$$

$$\cos R = \frac{14}{18} \doteq 0.7778$$

$$\tan R = \frac{12}{14} \doteq 0.8571$$

For $\angle T$,

$$\sin T = \frac{14}{18} \doteq 0.7778$$

$$\cos T = \frac{12}{18} \doteq 0.6667$$

$$\tan T = \frac{14}{12} \doteq 1.1667$$

b. For $\angle I$,

$$\sin I = \frac{15}{38} \doteq 0.3947$$

$$\cos I = \frac{35}{38} \doteq 0.9211$$

$$\tan I = \frac{15}{35} \doteq 0.4286$$

For $\angle P$,

$$\sin P = \frac{35}{38} \doteq 0.9211$$

$$\cos P = \frac{15}{38} \doteq 0.3947$$

$$\tan P = \frac{35}{15} \doteq 2.3333$$

3. a. $t^2 = a^2 + m^2$

$t^2 = 11^2 + 17^2$

$t^2 = 121 + 289$

$t^2 = 410$

$\sqrt{t^2} = \sqrt{410}$

$t \doteq 20$

For $\angle$A,

$\sin A = \frac{11}{20}$

$= 0.5500$

$\cos A = \frac{17}{20}$

$= 0.8500$

$\tan A = \frac{11}{17}$

$\doteq 0.6471$

For $\angle$M,

$\sin M = \frac{17}{20}$

$= 0.8500$

$\cos M = \frac{11}{20}$

$= 0.5500$

$\tan M = \frac{17}{11}$

$\doteq 1.5455$

b. $o^2 = p^2 - m^2$

$o^2 = 36^2 - 14^2$

$o^2 = 1296 - 196$

$o^2 = 1100$

$\sqrt{o^2} = \sqrt{1100}$

$o \doteq 33$

For $\angle$M,

$\sin M = \frac{14}{36}$

$\doteq 0.3889$

$\cos M = \frac{33}{36}$

$\doteq 0.9167$

$\tan M = \frac{14}{33}$

$\doteq 0.4242$

For $\angle$O,

$\sin O = \frac{33}{36}$

$\doteq 0.9167$

$\cos O = \frac{14}{36}$

$\doteq 0.3889$

$\tan O = \frac{33}{14}$

$\doteq 2.3571$

c. $l^2 = e^2 - d^2$

$l^2 = 73^2 - 39^2$

$l^2 = 5329 - 1521$

$l^2 = 3808$

$\sqrt{l^2} = \sqrt{3808}$

$l \doteq 62$

For $\angle$L,

$\sin L = \frac{62}{73}$

$\doteq 0.8493$

$\cos L = \frac{39}{73}$

$\doteq 0.5342$

$\tan L = \frac{62}{39}$

$\doteq 1.5897$

For $\angle$D,

$\sin D = \frac{39}{73}$

$\doteq 0.5342$

$\cos D = \frac{62}{73}$

$\doteq 0.8493$

$\tan D = \frac{39}{62}$

$\doteq 0.6290$

Extensions

1. a. $r^2 = d^2 - e^2$

$r^2 = \left(15\sqrt{7}\right)^2 - (5.2)^2$

$r^2 = 1575 - 27.04$

$r^2 = 1547.96$

$\sqrt{r^2} = \sqrt{1547.96}$

$r \doteq 39.3$

For $\angle$R,

$\sin R = \frac{39.3}{39.7}$

$\doteq 0.9899$

$\cos R = \frac{5.2}{39.7}$

$\doteq 0.1310$

$\tan R = \frac{39.3}{5.2}$

$\doteq 7.5577$

For $\angle$E,

$\sin E = \frac{5.2}{39.7}$

$\doteq 0.1310$

$\cos E = \frac{39.3}{39.7}$

$\doteq 0.9899$

$\tan E = \frac{5.2}{39.3}$

$\doteq 0.1323$

b. $o^2 = p^2 - m^2$

$o^2 = (13.2)^2 - \left(3\frac{1}{4}\right)^2$

$o^2 \doteq 174.24 - 10.56$

$o^2 \doteq 163.68$

$\sqrt{o^2} \doteq \sqrt{163.68}$

$o \doteq 12.8$

For $\angle$M,

$\sin M = \frac{3.25}{13.2}$

$\doteq 0.2462$

$\cos M = \frac{12.8}{13.2}$

$\doteq 0.9697$

$\tan M = \frac{3.25}{12.8}$

$\doteq 0.2539$

For $\angle$O,

$\sin O = \frac{12.8}{13.2}$

$\doteq 0.9697$

$\cos O = \frac{3.25}{13.2}$

$\doteq 0.2462$

$\tan O = \frac{12.8}{3.25}$

$\doteq 3.9385$

c. $c^2 = a^2 + b^2$

$c^2 = \left(3\frac{1}{2}\right)^2 + \left(4\frac{2}{3}\right)^2$

$c^2 = 12.25 + 21.78$

$c^2 \doteq 34.03$

$\sqrt{c^2} \doteq \sqrt{34.03}$

$c \doteq 5.8$

For $\angle$A,

$\sin A = \frac{3.5}{5.8}$

$\doteq 0.6034$

$\cos A = \frac{4.7}{5.8}$

$\doteq 0.8103$

$\tan A = \frac{3.5}{4.7}$

$\doteq 0.7447$

For $\angle$B,

$\sin B = \frac{4.7}{5.8}$

$\doteq 0.8103$

$\cos B = \frac{3.5}{5.8}$

$\doteq 0.6034$

$\tan B = \frac{4.7}{3.5}$

$\doteq 1.3429$

d. $n^2 = i^2 + s^2$

$n^2 = \left(6\frac{1}{3}\right)^2 + \left(10\sqrt{5}\right)^2$

$n^2 \doteq 40.11 + 500$

$n^2 \doteq 540.11$

$\sqrt{n^2} \doteq \sqrt{540.11}$

$n \doteq 23.2$

For $\angle S$,

$\sin S = \frac{22.4}{23.2}$

$\doteq 0.9655$

$\cos S = \frac{6.3}{23.2}$

$\doteq 0.2716$

$\tan S = \frac{22.4}{6.3}$

$\doteq 3.5556$

For $\angle I$,

$\sin I = \frac{6.3}{23.2}$

$\doteq 0.2716$

$\cos I = \frac{22.4}{23.2}$

$\doteq 0.9655$

$\tan I = \frac{6.3}{22.4}$

$\doteq 0.2813$

Exploring Topic 4

Activity 1

Determine the measure of an acute angle when two sides are known.

1. a. $\tan \theta = \frac{15}{7}$

$\tan \theta = 2.1429$

$\theta = 65°$

b. $\sin \theta = \frac{15}{26}$

$\sin \theta = 0.5769$

$\theta = 35°$

c. $\cos \theta = \frac{41}{73}$

$\cos \theta = 0.5616$

$\theta = 56°$

2. a. $\tan \theta = \frac{22}{6}$

$\tan \theta = 3.6667$

$\theta = 75°$

b. $\sin \theta = \frac{31}{73}$

$\sin \theta = 0.4247$

$\theta = 25°$

c. $\cos \theta = \frac{24}{56}$

$\cos \theta = 0.4286$

$\theta = 65°$

3. a. $\tan \theta = \frac{32}{14}$

$\tan \theta = 2.2857$

$\theta = 66°$

b. $\sin \theta = \frac{30}{75}$

$\sin \theta = 0.4000$

$\theta = 24°$

c. $\cos \theta = \frac{61}{103}$

$\cos \theta = 0.5922$

$\theta = 54°$

4. a. $\tan\theta = \frac{16}{9}$

$\tan\theta = 1.7778$

1.7778 is between 1.7321 and 1.8040 or between 60° and 61°.

Differences: $1.7778 - 1.7321 = 0.0457$
$1.8040 - 1.7778 = 0.0262$
1.7778 is closer to 1.8040, so $\theta = 61°$.

b. $\sin\theta = \frac{16}{25}$

$\sin\theta = 0.6400$

0.6400 is between 0.6293 and 0.6428 or between 39° and 40°.

Differences: $0.6400 - 0.6293 = 0.0107$
$0.6428 - 0.6400 = 0.0028$
0.6400 is closer to 0.6428, so $\theta = 40°$.

c. $\cos\theta = \frac{9}{25}$

$\cos\theta = 0.3600$

0.3600 is between 0.3584 and 0.3746 or 68° and 69°.

Differences: $0.3600 - 0.3584 = 0.0016$
$0.3746 - 0.3600 = 0.0146$
0.3600 is closer to 0.3584, so $\theta = 69°$.

Activity 2

Determine the length of a side when the measure of one side and one acute angle are known.

1. a. $\tan 16° = \frac{a}{14}$

$0.2867 = \frac{a}{14}$

$a = 14 \times 0.2867$

$a \doteq 4.01$

b. $\tan 36° = \frac{a}{42}$

$0.7265 = \frac{a}{42}$

$a = 42 \times 0.7265$

$a \doteq 30.51$

2. a. $\tan 39° = \frac{d}{24}$

$0.8098 = \frac{d}{24}$

$d = 24 \times 0.8098$

$d \doteq 19.44$

b. $\tan 69° = \frac{d}{52}$

$2.6051 = \frac{d}{52}$

$d = 52 \times 2.6051$

$d \doteq 135.47$

3. a. $\sin 22° = \frac{b}{54}$

$0.3746 = \frac{b}{54}$

$b = 54 \times 0.3746$

$b \doteq 20.23$

b. $\sin 32° = \frac{b}{24}$

$0.5299 = \frac{b}{24}$

$b = 24 \times 0.5299$

$b \doteq 12.72$

4. a. $\sin 62° = \frac{e}{84}$

$0.8829 = \frac{e}{84}$

$e = 84 \times 0.8829$

$e \doteq 74.16$

b. $\sin 82° = \frac{e}{107}$

$0.9903 = \frac{e}{107}$

$e = 107 \times 0.9903$

$e \doteq 105.96$

5. a. $\cos 47° = \frac{14}{c}$

$0.6820 = \frac{14}{c}$

$c = 14 \div 0.6820$

$c \doteq 20.53$

b. $\cos 87° = \frac{93}{c}$

$0.0523 = \frac{93}{c}$

$c = 93 \div 0.0523$

$c \doteq 1778.20$

6. a. $\cos 52° = \frac{33}{f}$

$0.6157 = \frac{33}{f}$

$f = 33 \div 0.6157$

$f \doteq 53.60$

b. $\cos 72° = \frac{49}{f}$

$0.3090 = \frac{49}{f}$

$f = 49 \div 0.3090$

$f \doteq 158.58$

Activity 3

Solve right triangles.

1. The measures of c, $\angle A$, and $\angle B$ must be found.

$c^2 = a^2 + b^2$

$c^2 = 39^2 + 23^2$

$c^2 = 1521 + 529$

$c^2 = 2050$

$\sqrt{c^2} = \sqrt{2050}$

$c \doteq 45$

$\sin A = \frac{39}{45}$

$\doteq 0.8667$

$\angle A = 60°$

$\cos B = \frac{39}{45}$

$\doteq 0.8667$

$\angle B = 30°$

Summary:

$\angle A = 60°$ $a = 39$

$\angle B = 30°$ $b = 23$

$\angle C = 90°$ $c = 45$

You may have obtained measures of 59° for $\angle A$ and 31° for $\angle B$ if you had used the tangent ratio.

2. The measures of $\angle D$, f, and d must be found.

$\tan 37° = \frac{52}{d}$

$0.7536 = \frac{52}{d}$

$d = 52 \div 0.7536$

$d \doteq 69$

$\sin 37° = \frac{52}{f}$

$0.6018 = \frac{52}{f}$

$f = 52 \div 0.6018$

$f \doteq 86$

$\angle D = 180° - (90° + 37°)$
$= 180° - 127°$
$= 53°$

Summary:

$\angle D = 53°$ $\quad d = 69$
$\angle E = 37°$ $\quad e = 52$
$\angle F = 90°$ $\quad f = 86$

3. The measures of $\angle O$, o, and m must be found.

$\angle O = 180° - (90° + 73°)$
$= 180° - 163°$
$= 17°$

$\sin 73° = \frac{m}{36}$ $\qquad$ $\cos 73° = \frac{o}{36}$

$0.9563 = \frac{m}{36}$ $\qquad$ $0.2924 = \frac{o}{36}$

$m = 36 \times 0.9563$ $\qquad$ $o = 36 \times 0.2924$

$m \doteq 34$ $\qquad$ $o \doteq 11$

Summary:

$\angle M = 73°$ $\quad m = 34$
$\angle O = 17°$ $\quad o = 11$
$\angle P = 90°$ $\quad p = 36$

4. The measures of $\angle A$, r, and t must be found.

$\angle A = 180° - (90° + 24°)$
$= 180° - 114°$
$= 66°$

$\tan 24° = \frac{r}{46}$

$0.4452 = \frac{r}{46}$

$r = 46 \times 0.4452$

$r \doteq 20$

$t^2 = r^2 + a^2$

$t^2 = 20^2 + 46^2$

$t^2 = 400 + 2116$

$t^2 = 2516$

$\sqrt{t^2} = \sqrt{2516}$

$t \doteq 50$

Summary:

$\angle A = 66°$ $\quad a = 46$
$\angle R = 24°$ $\quad r = 20$
$\angle T = 90°$ $\quad t = 50$

5. The measures of $\angle X$, x, and y must be found.

$\cos 56° = \frac{x}{109}$

$0.5592 = \frac{x}{109}$

$x = 109 \times 0.5592$

$x \doteq 61$

$\tan 56° = \frac{y}{61}$

$1.4826 = \frac{y}{61}$

$y = 61 \times 1.4826$

$y \doteq 90$

$\tan X = \frac{x}{y}$

$\tan X = \frac{61}{90}$

$\tan X = 0.6778$

$X = 34°$

Summary:

$\angle X = 34°$
$\angle Y = 56°$
$\angle Z = 90°$

$x = 61$
$y = 90$
$z = 109$

Activity 4

Solve word problems involving right triangles.

1. $\tan 57° = \frac{168}{d}$ The angle of elevation from the boat is also 57°.

$1.5399 = \frac{168}{d}$

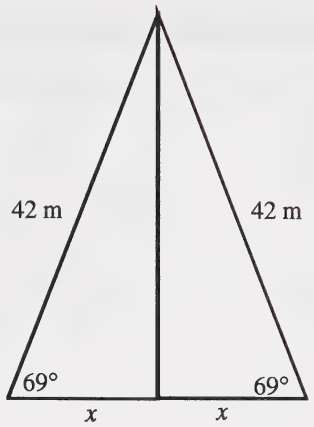
$d = 168 \div 1.5399$

$d \doteq 109.098$

$d \doteq 109$

The sailboat is approximately 109 m from the base of the cliff.

2.



$\cos 69° = \frac{x}{42}$

$0.3584 = \frac{x}{42}$

$x = 42 \times 0.3584$

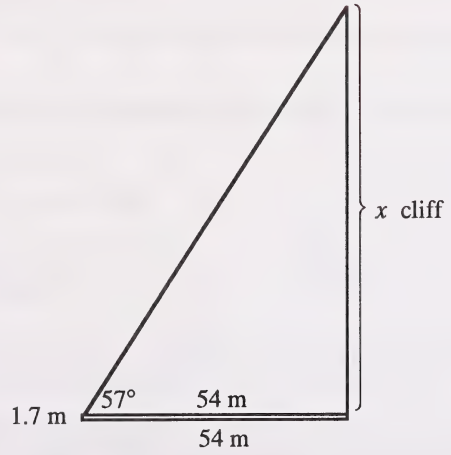
$x \doteq 15.05$

Two such distances would be $15.05 \times 2 \doteq 30.10$

$\doteq 30.1$

The approximate horizontal distance between the two wires along the ground is 30.1 m.

3.



Step 1: Find x.

$$\tan 57° = \frac{x}{54}$$

$$1.5399 = \frac{x}{54}$$

$$x = 54 \times 1.5399$$

$$x \doteq 83.15$$

$$x \doteq 83.2$$

Step 2: The height of the cliff is

$$83.2 \text{ m} + 1.7 \text{ m} = 84.9 \text{ m}.$$

4. $\sin 74° = \frac{16.3}{d}$

$$0.9613 = \frac{16.3}{d}$$

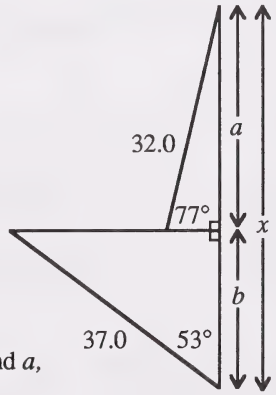
$$d = 16.3 \div 0.9613$$

$$d \doteq 16.956$$

$$d \doteq 17.0$$

The approximate length of the diagonal is 17.0 km.

5. Divide x into parts a and b.



To find a,

$$\sin 77° = \frac{a}{32}$$

$$0.9744 = \frac{a}{32}$$

$$a = 32 \times 0.9744$$

$$a \doteq 31.18$$

$$a \doteq 31.2$$

To find b,

$$\cos 53° = \frac{b}{37}$$

$$0.6018 = \frac{b}{37}$$

$$b = 37 \times 0.6018$$

$$b \doteq 22.26$$

$$b \doteq 22.3$$

The entire x-distance is 53.5 units. (31.2 + 22.3)

6. Find a first.

$$\sin 44° = \frac{a}{21.7}$$

$$0.6947 = \frac{a}{21.7}$$

$$a = 21.7 \times 0.6947$$

$$a \doteq 15.1$$

Find x next.

$$\sin 28° = \frac{15.1}{x}$$

$$0.4695 = \frac{15.1}{x}$$

$$x = 15.1 \div 0.4695$$

$$x \doteq 32.2$$

The approximate length of x is 32.2 units.

Then find the distance between A and C.

$$\tan 59° = \frac{AC}{56}$$

$$1.6643 = \frac{AC}{56}$$

$$AC = 56 \times 1.6643$$

$$AC \doteq 93$$

The distance x which stands for the width of the river is 51 m. (93 m – 42 m = 51 m)

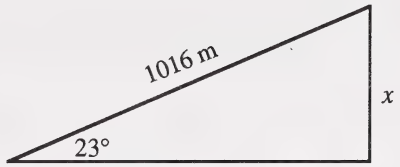
7. $$\sin 23° = \frac{x}{1016}$$

$$0.3907 = \frac{x}{1016}$$

$$x \doteq 1016 \times 0.3907$$

$$x \doteq 396.9512$$

$$x \doteq 397$$



At this point, their height above the horizontal is approximately 397 m.

8. Find the distance between D and C.

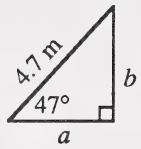
$$\tan 37° = \frac{42}{DC}$$

$$0.7536 = \frac{42}{DC}$$

$$DC = 42 \div 0.7536$$

$$DC \doteq 56 \text{ m}$$

9.



a. To find a, use

$$\cos 47° = \frac{a}{4.7}.$$

$$0.6820 = \frac{a}{4.7}$$

$$a = 4.7 \times 0.6820$$

$$a \doteq 3.2$$

The distance from the bottom of the ladder to the wall is 3.2 m.

b. To find b, use

$$\sin 47° = \frac{b}{4.7}.$$

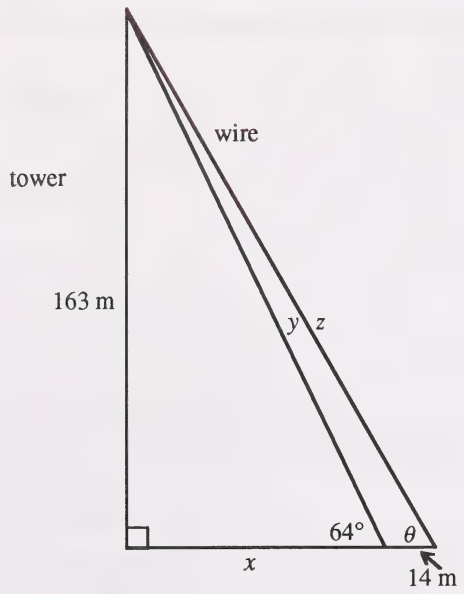
$$0.7314 = \frac{b}{4.7}$$

$$b = 4.7 \times 0.7314$$

$$b \doteq 3.4$$

The ladder reaches 3.4 m up the wall.

10.



a. Step 1: Find x.

$$\tan 64° = \frac{163}{x}$$

$$2.0503 = \frac{163}{x}$$

$$x = 163 \div 2.0503$$

$$x \doteq 79.5005$$

$$x \doteq 80$$

The value for x is 80 m.

Step 2: Find y.

$$\sin 64° = \frac{163}{y}$$

$$0.8988 = \frac{163}{y}$$

$$y = 163 \div 0.8988$$

$$y \doteq 181.353$$

$$y \doteq 181$$

The value for y is 181 m.

Step 3: Use $c^2 = a^2 + b^2$ to find z.

$$z^2 = (80+14)^2 + 163^2$$

$$z^2 = 94^2 + 163^2$$

$$z^2 = 8836 + 26\,569$$

$$z^2 = 35\,405$$

$$\sqrt{z^2} = \sqrt{35\,405}$$

$$z \doteq 188.162$$

$$z \doteq 188$$

The value for z is 188 m.

Step 4: Subtract y from z.
The new wire is 188 m – 181 m = 7 m longer.

b. Find θ.

$$\tan\theta = \frac{163}{80+14}$$

$$\tan\theta = \frac{163}{94}$$

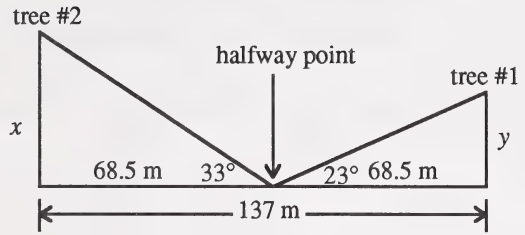
$$\tan\theta \doteq 1.7340$$

$$\theta \doteq 60.028$$

$$\theta \doteq 60°$$

The angle made between the new wire and the ground is approximately 60°.

11.



Step 1: Find x.

$$\tan 33° = \frac{x}{68.5}$$

$$0.6494 = \frac{x}{68.5}$$

$$x = 68.5 \times 0.6494$$

$$x \doteq 44.484$$

$$x \doteq 44$$

The height of tree #1 is approximately 44 m.

Step 2: Find y.

$$\tan 23° = \frac{y}{68.5}$$

$$0.4245 = \frac{y}{68.5}$$

$$y = 68.5 \times 0.4245$$

$$y \doteq 29.078$$

$$y \doteq 29$$

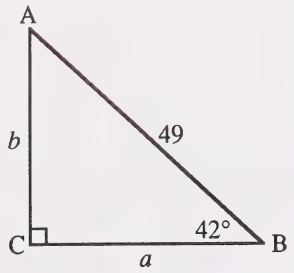
The height of tree #2 is approximately 29 m.

Step 3: Subtract y from x.

Tree #1 is 15 m taller than tree #2.
(44 m – 29 m = 15 m)

Extra Help

1. a.



The measures of $\angle A$, a, and b must be found.

$$\angle A = 180° - (90° + 42°)$$
$$= 180° - 132°$$
$$= 48°$$

$$\sin 42° = \frac{b}{49} \qquad \cos 42° = \frac{a}{49}$$

$$0.6691 = \frac{b}{49} \qquad 0.7431 = \frac{a}{49}$$

$$b = 49 \times 0.6691 \qquad a = 49 \times 0.7431$$

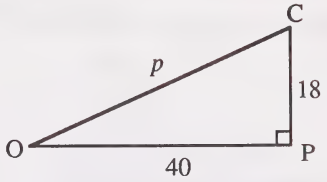
$$b = 32.7859 \qquad a = 36.4119$$

$$b \doteq 32.8 \qquad a \doteq 36.4$$

Summary:

$\angle A = 48°$ $\quad a = 36.4$
$\angle B = 42°$ $\quad b = 32.8$
$\angle C = 90°$ $\quad c = 49$

b.



The measures of $\angle C$, $\angle O$, and p must be found.

$$\tan O = \frac{18}{40} \qquad \tan C = \frac{40}{18}$$

$$\tan O = \frac{9}{20} \qquad \tan C = \frac{20}{9}$$

$$\tan O = 0.4500 \qquad \tan C = 2.2222$$

$$\angle O \doteq 24.228 \qquad \angle C \doteq 65.772$$

$$\angle O \doteq 24° \qquad \angle C \doteq 66°$$

$$\cos C = \frac{18}{p}$$

$$\cos 66° = \frac{18}{p}$$

$$0.4067 = \frac{18}{p}$$

$$p = 18 \div 0.4067$$

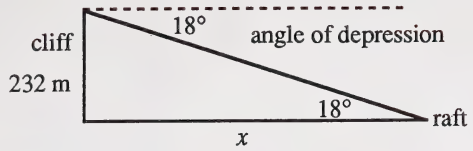
$$p \doteq 44.259$$

$$p \doteq 44$$

Summary:

$\angle C = 66°$ $\quad c = 40$
$\angle O = 24°$ $\quad o = 18$
$\angle P = 90°$ $\quad p = 44$

2.



Solve for x.

$$\tan 18° = \frac{232}{x}$$

$$0.3249 = \frac{232}{x}$$

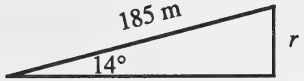
$$x = 232 \div 0.3249$$

$$x \doteq 714.066$$

$$x \doteq 714$$

To the nearest metre, the raft is 714 m from the base of the cliff.

3.



Solve for r.

$$\sin 14° = \frac{r}{185}$$

$$0.2419 = \frac{r}{185}$$

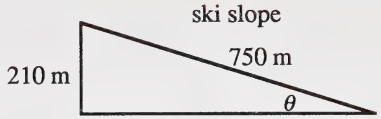
$$r = 185 \times 0.2419$$

$$r = 44.7515$$

$$r \doteq 45$$

The rise of the road under these conditions is approximately 45 m.

4.



Solve for θ.

$$\sin \theta = \frac{210}{750}$$

$$\sin \theta = 0.2800$$

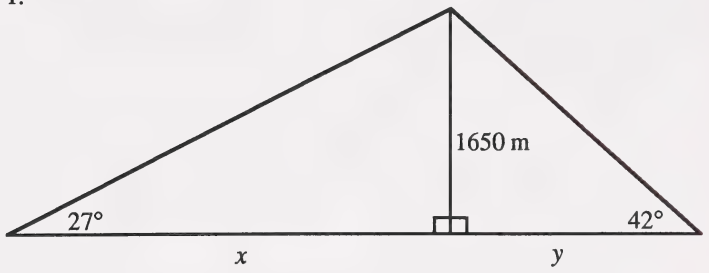
$$\theta \doteq 16.26$$

$$\theta \doteq 16°$$

The angle of elevation of the ski slope is 16° to the nearest degree.

Extensions

1.



Step 1: Solve for x.

$$\tan 27° = \frac{1650}{x}$$

$$0.5095 = \frac{1650}{x}$$

$$x = 1650 \div 0.5095$$

$$x \doteq 3238.469$$

$$x \doteq 3238$$

Step 2: Solve for y.

$$\tan 42° = \frac{1650}{y}$$

$$0.9004 = \frac{1650}{y}$$

$$y = 1650 \div 0.9004$$

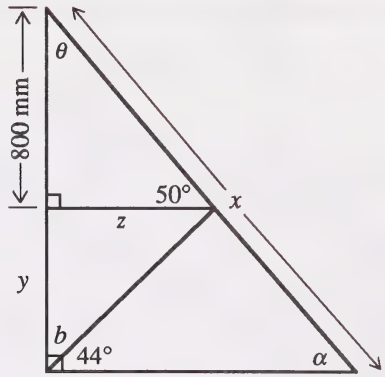
$$y \doteq 1832.519$$

$$y \doteq 1833$$

Step 3: Find the sum of x and y.

The total length of the tunnel to the nearest metre is 5071 m. (3238 + 1833 = 5071)

2.



Step 1: Find θ.

$$\theta = 180° - (90° + 50°)$$
$$\theta = 180° - 140°$$
$$\theta = 40°$$

Step 2: Find α.

$$\alpha = 180° - (90 + 40°)$$

$$\alpha = 180° - 130°$$

$$\alpha = 50°$$

Step 3: Find b.

$$b = 90° - 44°$$
$$b = 46°$$

Step 4: Find z.

$$\tan 50° = \frac{800}{z}$$

$$1.1918 = \frac{800}{z}$$

$$z = 800 \div 1.1918$$

$$z \doteq 671.254$$

$$z \doteq 671$$

Step 5: Find y.

$$\tan b = \frac{z}{y}$$

$$\tan 46° = \frac{671}{y}$$

$$1.0355 = \frac{671}{y}$$

$$y = 671 \div 1.0355$$

$$y \doteq 647.996$$

$$y \doteq 648$$

Step 6: Add y to 800.

$$y + 800 = 648 + 800$$
$$= 1448$$

Step 7: Find x using the sine ratio.

$$\sin \alpha = \frac{800 + y}{x}$$

$$\sin 50° = \frac{1448}{x}$$

$$0.7660 = \frac{1448}{x}$$

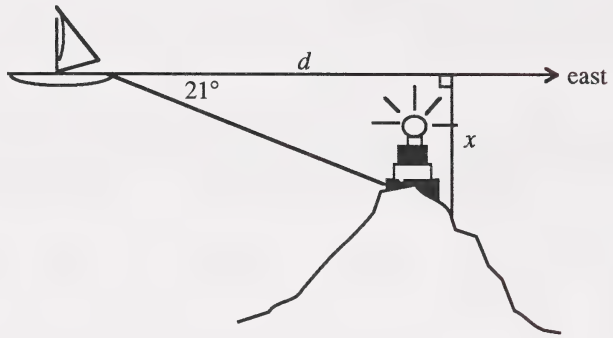
$$x = 1448 \div 0.7660$$

$$x \doteq 1890.339$$

$$x \doteq 1890 \text{ mm}$$

The value for x is 1890 mm, 189 cm, or 1.89 m.

3.



Find d first. Let r = speed and t = time.

$$d = rt$$

$$d = 18.4 \times 0.75$$

$$d = 13.8$$

The ship goes a distance of 13.8 km before it is directly opposite the lighthouse.

Solve for x.

$$\tan 21° = \frac{x}{13.8}$$

$$0.3839 = \frac{x}{13.8}$$

$$x = 13.8 \times 0.3839$$

$$x \doteq 5.298$$

$$x \doteq 5.3$$

When directly opposite the lighthouse, the ship is approximately 5.3 km from the shore and the lighthouse.

Appendix B

Sines, Cosines, and Tangents 0° – 90°

Degrees	Sine	Cosine	Tangent
0°	0.0000	1.0000	0.0000
1°	0.0175	0.9998	0.0175
2°	0.0349	0.9994	0.0349
3°	0.0523	0.9986	0.0524
4°	0.0698	0.9976	0.0699
5°	0.0872	0.9962	0.0875
6°	0.1045	0.9945	0.1051
7°	0.1219	0.9925	0.1228
8°	0.1392	0.9903	0.1405
9°	0.1564	0.9877	0.1584
10°	0.1736	0.9848	0.1763
11°	0.1908	0.9816	0.1944
12°	0.2079	0.9781	0.2126
13°	0.2250	0.9744	0.2309
14°	0.2419	0.9703	0.2493
15°	0.2588	0.9659	0.2679
16°	0.2756	0.9613	0.2867
17°	0.2924	0.9563	0.3057
18°	0.3090	0.9511	0.3249
19°	0.3256	0.9455	0.3443
20°	0.3420	0.9397	0.3640
21°	0.3584	0.9336	0.3839
22°	0.3746	0.9272	0.4040
23°	0.3907	0.9205	0.4245
24°	0.4067	0.9135	0.4452
25°	0.4226	0.9063	0.4663
26°	0.4384	0.8988	0.4877
27°	0.4540	0.8910	0.5095
28°	0.4695	0.8829	0.5317
29°	0.4848	0.8746	0.5543
30°	0.5000	0.8660	0.5774
31°	0.5150	0.8572	0.6009
32°	0.5299	0.8480	0.6249
33°	0.5446	0.8387	0.6494
34°	0.5592	0.8290	0.6745
35°	0.5736	0.8192	0.7002
36°	0.5878	0.8090	0.7265
37°	0.6018	0.7986	0.7536
38°	0.6157	0.7880	0.7813
39°	0.6293	0.7771	0.8098
40°	0.6428	0.7660	0.8391
41°	0.6561	0.7547	0.8693
42°	0.6691	0.7431	0.9004
43°	0.6820	0.7314	0.9325
44°	0.6947	0.7193	0.9657
45°	0.7071	0.7071	1.0000

Degrees	Sine	Cosine	Tangent
45°	0.7071	0.7071	1.0000
46°	0.7193	0.6947	1.0355
47°	0.7314	0.6820	1.0724
48°	0.7431	0.6691	1.1106
49°	0.7547	0.6561	1.1504
50°	0.7660	0.6428	1.1918
51°	0.7771	0.6293	1.2349
52°	0.7880	0.6157	1.2799
53°	0.7986	0.6018	1.3270
54°	0.8090	0.5878	1.3764
55°	0.8192	0.5736	1.4281
56°	0.8290	0.5592	1.4826
57°	0.8387	0.5446	1.5399
58°	0.8480	0.5299	1.6003
59°	0.8572	0.5150	1.6643
60°	0.8660	0.5000	1.7321
61°	0.8746	0.4848	1.8040
62°	0.8829	0.4695	1.8807
63°	0.8910	0.4540	1.9626
64°	0.8988	0.4384	2.0503
65°	0.9063	0.4226	2.1445
66°	0.9135	0.4067	2.2460
67°	0.9205	0.3907	2.3559
68°	0.9272	0.3746	2.4751
69°	0.9336	0.3584	2.6051
70°	0.9397	0.3420	2.7475
71°	0.9445	0.3256	2.9042
72°	0.9511	0.3090	3.0777
73°	0.9563	0.2924	3.2709
74°	0.9613	0.2756	3.4874
75°	0.9659	0.2588	3.7321
76°	0.9703	0.2419	4.0108
77°	0.9744	0.2250	4.3315
78°	0.9781	0.2079	4.7046
79°	0.9816	0.1908	5.1446
80°	0.9848	0.1736	5.6713
81°	0.9877	0.1564	6.3138
82°	0.9903	0.1392	7.1154
83°	0.9925	0.1219	8.1443
84°	0.9945	0.1045	9.5144
85°	0.9962	0.0872	11.4301
86°	0.9976	0.0698	14.3007
87°	0.9986	0.0523	19.0811
88°	0.9994	0.0349	28.6363
89°	0.9998	0.0175	57.2900
90°	1.0000	0.0000	undefined

NOTES

NOTES

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